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VII.—ELECTRO-ENDOSMOSIS AND ELECTROLYTIC WATER TRANSPORT.

By H. C. HEPBURN, *B.Sc.*, Research Student, Birkbeck College.

Received September 25, 1926.

(*Communicated by Prof.* A. GRIFFITHS, *D.Sc.*)

ABSTRACT.

This Paper gives the results of determinations of liquid transport produced by passing an electric current through aqueous solutions of copper sulphate divided perpendicular to the flow of electricity by a diaphragm of powdered glass. The probable factors that determine the liquid transport are investigated, over a wide range of concentrations, by an examination of the dependence of the flow at constant applied voltage and of the electric charge of the diaphragm on the electrolyte concentration, and also by a study of the relation between the flow per faraday and the dilution of the electrolyte.

INTRODUCTION.

THE nature of the liquid transport which occurs when an electric current is passed through an electrolyte, divided by means of a diaphragm perpendicular to the flow of electricity, has been investigated by Remy[1] over a wide range of concentrations. Remy has shown, by means of a comparative study of data obtained with several electrolytes and with diaphragms of various mateıials, that two factors fall to be considered: (*a*) electro-endosmosis—i.e., the surface phenomenon which is explained on the classical theory of Helmholtz[2] in terms of the electrical double layer; and (*b*) electrolytic water transport—i.e., transport of water produced by the motion of the ions of the electrolyte, the latter factor becoming important at the higher concentrations.

The present work consists of a series of measurements with a diaphragm of powdered glass and aqueous solutions of copper sulphate over the concentration range 0·0002 normal to 1·0 normal; and the nature of the liquid transport is investigated by an examination of the dependence of the flow at constant applied voltage, and of the electric charge of the diaphragm on the electrolyte concentration, and also by a study of the relation between the flow per faraday and the dilution of the electrolyte. The line of treatment adopted appears to present several novel features; and a simple relation is obtained, at the higher concentrations, between the liquid flow per faraday and the dilution of the electrolyte, which, so far as can be traced from a search of the literature, has not previously been recorded.

THE APPARATUS.

The apparatus employed was the same as that used by the author in a previous work,[3] except that copper electrodes were used in place of electrodes of silver and silver chloride.

The porous diaphragm consisted of a quantity of easily fusible glass in the form of a 200 mesh powder, securely held in position in the body of the apparatus by means of glass wool plugs. The glass powder was carefully purified as previously described.[3]

The electrodes consisted of lengths of copper wire, 1 mm. in diameter, wound into the form of flat spirals and fixed in the apparatus with the plane of each spiral normal to the axis of the diaphragm.

Experimental.

The liquid flow was measured, as in the previous work, by timing a short air bubble over 5 cm. of a calibrated tube forming part of the circuit of the apparatus, the initial determination being with conductivity water. Readings of current and applied potential difference were taken at the beginning and end of the period of measurement of liquid transport. The apparatus was next emptied, and the dia-

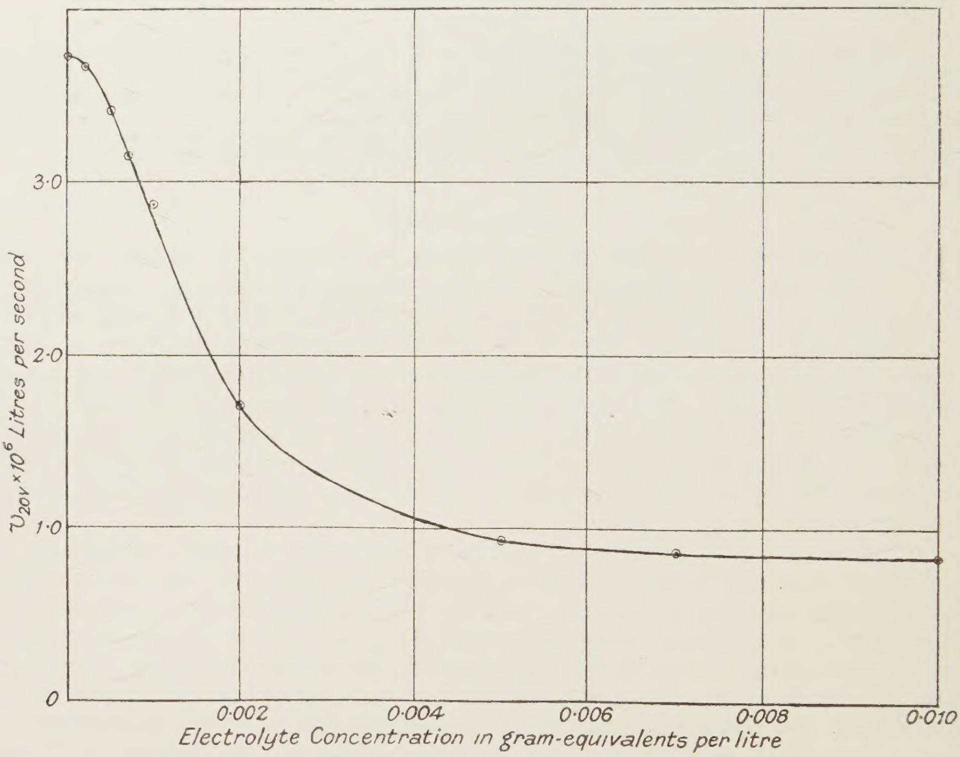

Fig. 1.—Liquid Transport at Constant Applied Voltage.

phragm thoroughly washed out with a considerable quantity of 0·0002 normal copper sulphate solution, following the method previously described by the author.(3) The apparatus was then allowed to stand for about 30 minutes, and a further series of observations made of the current, applied potential difference and time of liquid flow. In a similar manner, measurements were made with solutions of increasing concentration up to 1·0 normal.

The current was reversed after each determination so as to bring the electrodes, as far as possible, back to their original state. Control experiments by the author have shown that the initial determination, at each concentration, is in general the

most accurate, and for this reason measurements on reversal of the current are not given.

The temperature of the room in which the work was undertaken was maintained, as far as possible, constant at 18°C., determinations being made when the solution in the apparatus attained this temperature. At the higher concentrations, however, the current readings indicated a certain amount of heating in the diaphragm. To ascertain the effect of this and other possible disturbing factors, control experiments were carried out with several solutions, within the concentration range of the present work, by timing the bubble over alternate sections of the calibrated tube. The liquid transport, at a given concentration, was found to be approximately proportional to the mean current in each interval, the results indicating that the values given in column 5 of Table I for the liquid transport per faraday exceed the values corresponding to a temperature of 18°C. by about 3 per cent. for 0·5 normal solution, 1·6 per cent. for 0·2 normal solution, and by under 1 per cent. for the lower concentrations.

Results.

(*a*) *Liquid Transport at Constant Applied Voltage.*

The results are given in Table I and shown graphically (up to concentration 0·10 normal) in Fig. 1.

Table I.

c.	V.	T.	$v_{20v} \times 10^6$	v_f
Gram. equivs. per litre.	Volts.	Secs.	Litres per sec.	Litres.
0·00	20·50	136·8	3·744	759·0
0·0002	20·76	137·5	3·679	496·3
0·0005	20·18	152·4	3·416	324·1
0·0007	20·18	165·0	3·154	272·2
0·001	20·02	182·2	2·879	202·5
0·002	20·02	306·3	1·713	72·1
0·005	20·20	554·7	0·937	18·34
0·007	20·26	595·8	0·870	12·85
0·01	20·26	614·0	0·844	9·32
0·02	20·22	697·6	0·745	4·754
0·05	20·22	703·7	0·738	2·276
0·1	20·22	735·3	0·706	1·238
0·2	20·02	787·4	0·666	0·680
0·5	20·04	756·7	0·693	0·330

c in Table is the concentration of the solution ; V the potential difference applied at the electrodes ; T the time of flow over 5 cm. of the bubble tube and corresponding to a liquid transport of 0·525 c.c. ; and v_{20v} the liquid transport for an applied potential difference of 20 volts given by $\frac{0{\cdot}525}{10^3} \cdot \frac{1}{T} \cdot \frac{20}{V}$.

With diaphragm 5 cm. in length and electrodes 5·8 cm. apart, as in the present work, the author has shown previously[3] that E, the total fall in potential through the diaphragm, is given, within a fraction of 1 per cent., by Vl_d/l_e, where l_d is the length of the diaphragm and l_e the distance between the electrodes ; the values of v_{20v} in column 4 of Table I thus relate to a constant value of E of 17·24 volts.

The direction of liquid flow was from anode to cathode for all concentrations up to 0·5 normal; no perceptible flow was obtained with 1·0 normal solution.

(*b*) *Liquid Transport per Faraday.*

Values of v_f, the liquid transport per faraday, are given in column 5 of Table I. The values are based upon the mean current flowing through the diaphragm during each period of measurement of liquid flow (*see* Table II, column 3).

(*c*) *The Charge of the Diaphragm.*

Smoluchowski[4] has shown, from theoretical considerations, that electro-endosmosis must increase the ordinary voltaic conductivity in consequence of the surface current produced. His result, which relates to the case of a single capillary, is conveniently given in the form $i_\omega = \rho v/q$. i_ω is the surface current, ρ the charge per unit length and q the cross-section of the capillary, and v the liquid transport in unit time. The corresponding expression for a battery of capillaries or porous diaphragm is $i_\omega = Pv/lq$, where P is the total charge and q and l the effective cross-section and length of the diaphragm, giving $P = lq \,.\, i_\omega/v$.

In Table II, I_i is the initial total current flowing through the diaphragm as measured during each determination of liquid transport. The values given correspond to the room temperature—i.e., 18°C.—and are in general smaller than the values of the mean current (I_m, column 3); at concentrations not greater than 0·01 normal, however, the differences between the two quantities are inappreciable.

Column 5 of Table II gives the ratio of the initial total current ($I_i \,.\, 20/V$), corresponding to an applied potential difference of 20 volts, to the specific conductivity (λ) of the solution, taking the values of λ (column 4 of Table) from the data of Kohlrausch. The values of the ratio over the concentration range 0·005 normal to 1·0 normal differ in general from the mean value for these concentrations by less

Table II.

c.	$I^i \times 10^3$	$I_m \times 10^3$	$\lambda \times 10^3$	$\frac{I_i}{\lambda} \cdot \frac{20}{V}$	$i_\omega \times 10^3$	P
Gram. equivs. per litre.	Amperes.		Recip. ohms.	—	Amperes.	Cou-lombs.
0·00	0·494	0·495	$1\text{–}2 \times 10^{-6}$	483–241	0·471–0·459	
					Mean 0·465	2·19
0·0002	0·741	0·743	0·02159	33·06	0·454	2·17
0·0005	1·024	1·027	0·0518	19·59	0·388	2·00
0·0007	1·126	1·129	0·0710	15·71	0·257	1·43
0·001	1·372	1·374	0·0986	13·90	0·176	1·08
0·002	2·291	2·295	0·1839	12·45	0·061	0·63
0·005	4·978	4·983	0·4049	12·17		
0·007	6·58	6·62	0·537	12·09		
0·01	8·81	8·86	0·717	12·13		
0·02	15·13	15·29	1·248	11·99		
0·05	31·17	31·65	2·558	12·05		
0·1	54·6	55·7	4·385	12·31		
0·2	91·3	94·7	7·532	12·12		
0·5	188·8	202·9	15·40	12·24		
1·0	314·0	...	25·77	12·18		

than 1 per cent. Considering the nature of the diaphragm, such differences are probably no greater than those due to the slight variations in the piling of the glass powder; and the ratio over the concentration range mentioned may reasonably be taken as constant. This result, considered in conjunction with the fact that no liquid transport was perceptible in 1·0 normal solution, indicates that the observed current over the concentration range 0·005 normal to 1·0 normal was practically wholly voltaic—i.e., due almost entirely to the motion of the ions of the electrolyte. The mean value of the ratio over the concentration range mentioned is 12·14, giving the voltaic current for an applied potential difference of 20 volts, and for a total fall in potential through the diaphragm of 17·24 volts (*see* page 101) as $12{\cdot}14\lambda$.

The surface current i_ω (Table II, column 6) is given by the difference between the total current and the voltaic current—i.e., by $I_m \frac{20}{V} - 12{\cdot}14\lambda$—for an applied potential difference of 20 volts. The use of the conductivity data of Kohlrausch in calculating the voltaic current for the solutions of concentration less than 0·005 normal has previously been justified by the author(3) in control experiments embodying conditions similar to those obtaining in the present work.

The value of lq, required to calculate P, the charge of the diaphragm, is obtained as follows:—

It is shown above that the voltaic current for a total fall in potential (E) through the diaphragm of 17·24 volts is given by $12{\cdot}14\lambda$; the voltaic current is also given by $E\lambda q/l$, where q and l are respectively the effective cross-section and length of the diaphragm. Equating these two quantities, we have $q/l = 12{\cdot}14/17{\cdot}24$, whence $ql = 17{\cdot}60$, taking the effective length of the diaphragm as 5·0 cm.

Values of P, the charge of the diaphragm, given by $lq \, . \, i_\omega/v$, are tabulated in column 7 of Table II, and shown graphically in Fig. 2. The considerations given in paragraph 3 of this Section indicate that the surface current, and consequently the charge of the diaphragm also, are negligible or zero at concentrations 0·005 normal and above.

Discussion of Results.

It will be seen from Fig. 1 and Table I (column 4) that the liquid transport for a constant applied potential difference across the diaphragm decreases strongly and regularly with the electrolyte concentration up to 0·005 normal—a result that is characteristic of electro-endosmosis—but thereafter, over the concentration range 0·005 normal to 0·5 normal, is not appreciably changed. At the same time the charge P of the diaphragm, as indicated in Fig. 2 and Table II (column 7), decreases with the concentration to a negligible or zero value at concentrations 0·005 normal and above. The values of P given in Table II may require modification if the liquid transport at concentrations below 0·005 normal is not due wholly to electro-endosmosis; but the result that P is negligible or zero at concentrations 0·005 normal and above remains unaffected. The foregoing results, and especially that relating to the charge of the diaphragm, indicate that the liquid transport with the solutions of concentrations 0·005 normal and above is due, in the main, to some factor other than electro-endosmosis.

It will be observed from Fig. 3, within the limit of experimental error, that the liquid transport per faraday, for solutions over the concentration range 0·005 normal to 0·1 normal is related to the dilution of the electrolyte by a linear law, but decreases

somewhat more rapidly from concentration 0·1 normal (to a zero or negligible value in 1·0 normal solution) than the linear law requires ; at concentrations below 0·005 normal, however, there appears to be no simple relation between the two quantities mentioned. This result, although not recorded by Remy, finds a parallel in his data[1] for a diaphragm of parchment paper and aqueous solutions of potassium chloride. Remy's values for potassium chloride solutions in the concentration

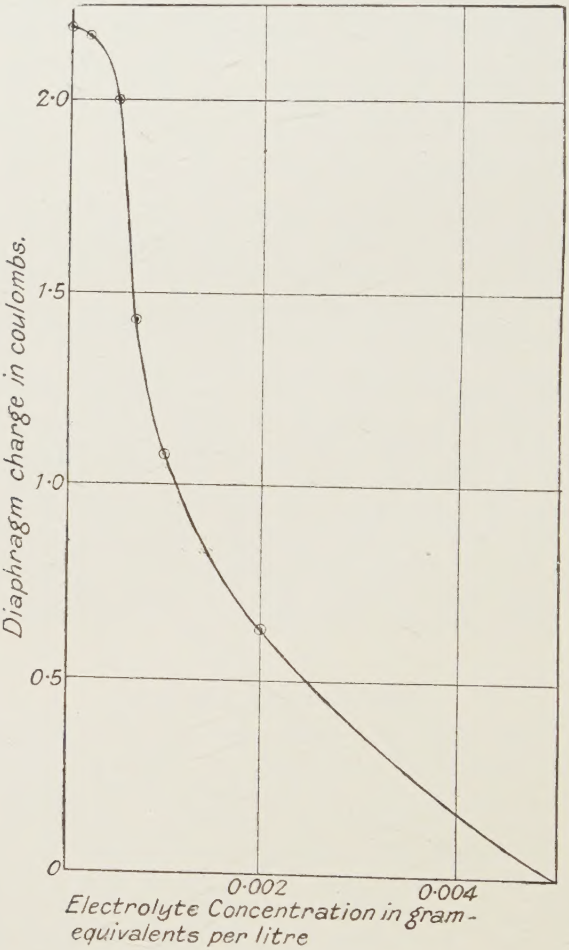

Fig. 2.—The Charge of the Diaphragm.

range 0·005 normal to 1·0 normal are plotted also in Fig. 3, and give an accurate linear relation, up to concentration 0·1 normal, between the liquid transport per faraday and the dilution ; the liquid flow for 1·0 normal solution is less than that required by the linear law.

Remy[1] observes from his results that parchment paper appears to form an ideal partition for the measurement of electrolytic water transport, and that the

electro-endosmotic effect with this material, even in high dilution, is very small. The relation observed between the liquid transport per faraday and the dilution in the case of the copper sulphate solutions of concentrations 0·005 normal and above thus appears to be characteristic of electrolytic water transport ; and at the concentrations mentioned the electric current flowing through the diaphragm appears to be due practically entirely to the motion of the ions of the electrolyte (*see* page 103).

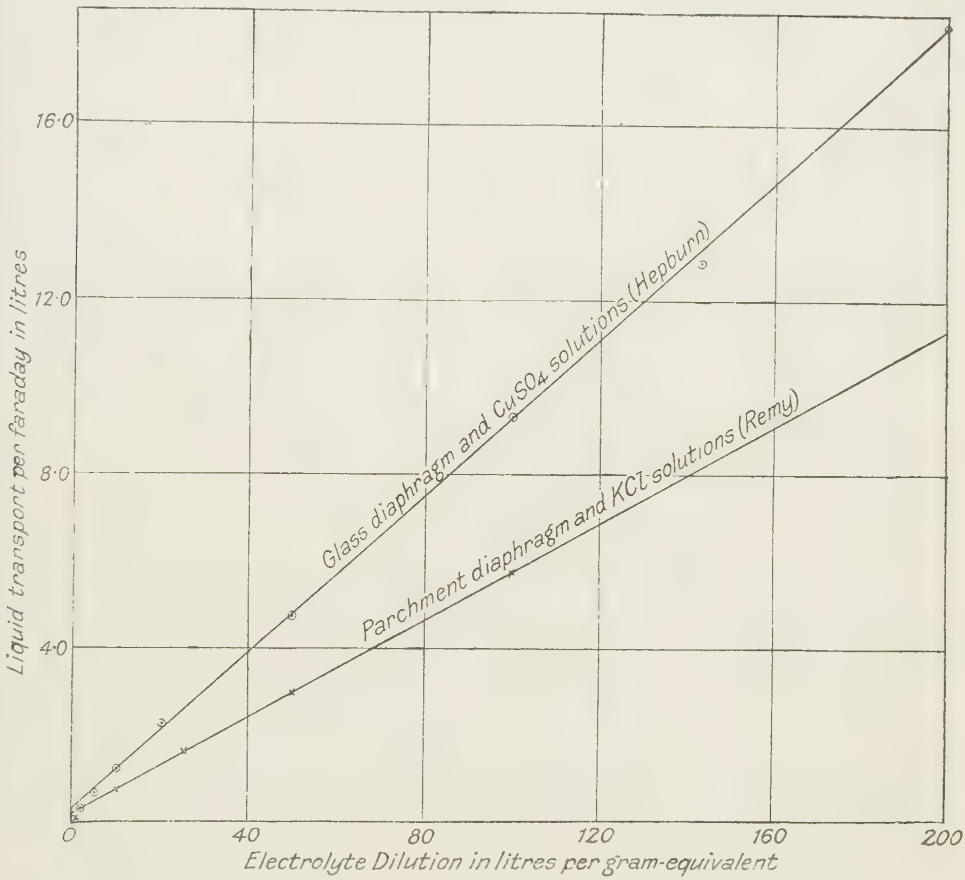

FIG. 3.—LIQUID TRANSPORT PER FARADAY.
(The ordinates represent tenths of a litre as regards Remy's values.)

SUMMARY OF RESULTS.

(1) The liquid transport for a constant applied potential difference, with solutions of copper sulphate in a diaphragm of powdered glass, decreases strongly and regularly with the concentration up to 0·005 normal, but thereafter, over the concentration range 0·005 normal to 0·5 normal, is not appreciably changed.

(2) The electric charge of the diaphragm decreases with the concentration to a negligible or zero value with solutions of concentrations 0·005 normal and above.

(3) The liquid transport per faraday, with solutions over the concentration

range 0·005 normal to 0·1 normal, is related to the dilution of the electrolyte by a linear law, and decreases to a zero or negligible value in 1·0 normal solution. Although not recorded by Remy, a similar result is given by his data for a diaphragm of parchment paper and aqueous solutions of potassium chloride, and with such a diaphragm the liquid transport, even in high dilution, appears to be mainly electrolytic.

(4) With the solutions of concentrations 0·005 normal and above, the electric current flowing through the diaphragm appears to be due practically entirely to the motion of the ions of the electrolyte.

The results of the present work thus indicate, for the case of solutions of copper sulphate in a diaphragm of powdered glass, that electro-endosmosis, with increasing electrolyte concentration, becomes a negligible factor at concentration 0·005 normal; and the data for the solutions of concentrations 0·005 normal and above appear to be characteristic of electrolytic water transport—i.e., transport of water produced by the motion of the ions of the electrolyte.

The author desires to thank Prof. A. Griffiths for facilities provided and for the interest he has taken in this investigation.

References.

(1) Remy, Zeit. Elektrochem., 29, 365 (1923).
(2) Helmholtz, Wied. Ann., 7, 337 (1879).
(3) Hepburn, Proc. Phys. Soc., 38, 363 (1926).
(4) Smoluchowski, Anz. Akad. Wiss Krakau, A, 182 (1903); Physik. Zeit., 6, 529 (1905).

DISCUSSION.

Dr. D. Owen pointed out that in effect the author measures the difference between the pressures on the two sides of his apparatus, and inquired whether there is an upper limit to this pressure difference corresponding to the horizontal part of the curve in Fig. 1. As regards the statement that the water transport per faraday is proportional to the dilution, this proportionality would seem to imply that with pure water the rate would become infinite.

Prof. F. L. Hopwood said that the data actually observed included the drift of the bubble in the capillary tube, and asked whether the bubble itself was subject to a potential gradient, which would produce a drift apart from that due to the mechanical difference of pressure. In addition to the two causes of water transport mentioned by the author, another possible factor was an action analogous to that which takes place in the Wehnelt interruptor, the rapid opening and closing of a capillary orifice by a bubble, when the current density is high, being capable of producing liquid transport.

Mr. L. Hartshorn: The author's apparatus may be regarded as a model illustrating the behaviour of certain industrial insulating materials, such as paper, and fibrous materials generally. These materials absorb water from the atmosphere, and, as Evershed* has pointed out, exhibit the phenomenon of water transport when subjected to the action of an electric field. One result of this is that the current through such materials is not directly proportional to the applied voltage. The charge on the diaphragm calculated by the author becomes the charge on the dielectric material in the analogous case in which a sheet of insulating material is placed between two metal plates, across which a constant potential difference is maintained, and in this case, since the material can be easily removed, it might be possible to measure the charge directly. Lübben† has recorded that sheets of telephone cable paper are observed to be charged when treated in this manner.

Modern work on industrial dielectrics suggests that many of them may be regarded as electrolytes, in which the ionic mobility is low, and this, again, suggests that the properties of the

* Evershed, Jour. I.E.E., Vol. 52, p. 51.

† Lübben, Arch. fur Elektrotechnik., Vol. 10, p. 283 (1910).

author's apparatus might give us some insight into the complex actions occurring in an imperfect dielectric.

Mr. A. R. PEARSON said that a further industrial problem connected with the subject of the Paper arose from the tendency of linseed paint to peel off, when wet, from an electrically-charged surface. This behaviour must be ascribed to the water transport which takes place through the film of oxidized linseed oil, the transport being in the same direction as the current in an alkaline medium, but reversible by acidulation. The author might think it worth while to investigate such a film for its industrial interest.

AUTHOR'S reply: In regard to the point raised by Dr. Owen, I may say that the difference in the liquid pressures on the two sides of the apparatus attains a steady value soon after the current is switched on, the value in each case being dependent upon the liquid transport through the diaphragm. In the present work the changes in the liquid levels were almost imperceptible. If, however, the bubble tube is interrupted, the pressure difference rises until equilibrium is reached, when the transport ceases. The proportionality between dilution and liquid transport fails for concentrations below 0·005 normal.

In reply to Prof. Hopwood, I should like to point out that the internal diameter of the bubble tube was about 3 mm.; the bubble was about 1 cm. in length, and, apart from the menisci, occupied the whole section of the tube. A control experiment described by Briggs, Bennett and Pierson,* who devised the form of apparatus employed in the present work, indicates that the potential gradient along the bubble tube produces no perceptible motion of the bubble, even when the potential difference applied at the electrodes is as great as 110 volts.

I am very grateful for the suggestions that have been made as to the possibility of extending the work to industrial problems.

* Jour. Phys. Chem., 22, 262 (1918).

VIII.—THE INPUT IMPEDANCES OF THERMIONIC VALVES AT LOW FREQUENCIES.

By L. HARTSHORN, *A.R.C.S.*, *B.Sc.*, *D.I.C.*, Electrical Department, National Physical Laboratory.

Received October 2, 1926.

ABSTRACT.

It is shown that accurate measurements of input admittance (or of input impedance) under various conditions can be made by means of the Schering Capacity Bridge. The input circuit is regarded as being equivalent to a condenser with a definite phase angle, φ, or "loss angle," $\delta = 90° - \varphi$, and the results are expressed by stating the effective capacity and value of $\tan \delta$ for each set of experimental conditions.

A series of measurements made on an *R* valve is recorded, and it is shown that the results are in good agreement with the theoretical investigations of Miller and Nichols. The theoretical investigation has been extended to allow for the effect of dielectric losses in the valve, since these were found by experiment to be rather large, and to have an appreciable effect on the capacity and phase angle of the input circuit.

It is shown that the effective capacity may vary from about $10 \mu\mu$F. to $100 \mu\mu$F. for an *R* valve, and the phase angle may vary from about 80° leading to 126° leading, depending mainly on the load in the anode circuit. Values of phase angle greater than 90° correspond to a negative resistance, or negative power factor, and occur when the load in the anode circuit is inductive. The variations of input capacity and phase angle with filament voltage, anode voltage, input voltage, and frequency are also investigated.

THE input impedance of a thermionic valve has been investigated theoretically by Miller* and Nichols,† who showed :—

1. That, owing to the effect of the capacity between the grid and anode, the input impedance varies with the load in the anode circuit.

2. That the input impedance cannot in general be considered as a pure capacity, even when a large negative grid bias is used. It may be considered as a capacity in series with a resistance, or as a condenser with a power factor, which is often quite large, and may be negative when the anode circuit contains an inductive load.

Miller made measurements of the input capacity at audio frequencies, when the load in the anode circuit consisted of a pure resistance, and the results showed that his formula for the input capacity was correct. He was not, however, able to check the formula for the equivalent series resistance, since he found that this was largely due to dielectric losses in the glass of the valve, in the cases he was investigating, and these losses had been neglected by him in obtaining the formulæ.

Morecroft‡ has made some measurements of input capacity and conductance at a frequency of 500 kilocycles, with both pure resistance and inductive loads in the anode circuit, though he did not attempt a complete analysis of the circuits.

In working with the Schering Bridge at audio frequencies, it was found possible to make measurements of the impedances of valves with considerable precision, so that it seemed to be worth while to attempt to verify the theoretical equations for

* Miller, Bull. Bureau Stds., Vol. 15, p. 367 (1919).

† Nichols, Phys. Rev., Vol. 13, p. 411 (1919).

‡ Morecroft, Proc. Inst. Radio Eng., Vol. 8, p. 239 (1920) ; or, "The Principles of Radio Communication," p. 428 (1921).

input impedance for the important case of an inductive load in the anode circuit. Since the electrode capacities were found to be by no means pure, the theoretical treatment was first extended to include the effects of dielectric losses in the valve.

Theoretical Treatment.

Following Nichols, the valve circuit may be represented by Fig. 1. Here F, G and P represent the filament, grid and plate respectively. The inter-electrode capacities are C_1, C_2 and C_3, and these are not considered to be pure, but to possess loss angles δ_1, δ_2 and δ_3 respectively, or, alternatively, there is associated with C_1 a conductance G_1, such that $\frac{G_1}{C_1\omega}=\tan\,\delta_1$, and so on. Z_1 represents the external impedance in the grid circuit, and Z_2 is that in the anode circuit. An E.M.F. e_1 is introduced into the grid circuit, and as a consequence we have the mesh currents

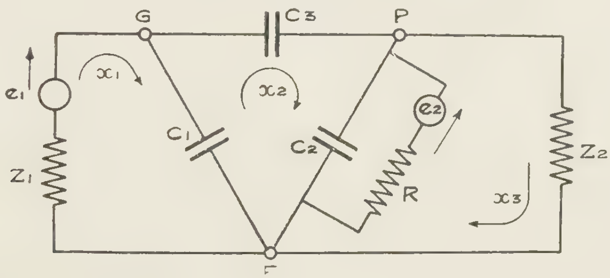

Fig. 1.—The Equivalent Network of the Valve.

x_1, x_2, x_3, in the network. The P.D. between the grid and filament is evidently $\frac{x_1-x_2}{\alpha_1}=\delta v$, where α_1 represents the conductance operator of the imperfect condenser C_1, i.e., $\alpha_1=G_1+jC_1\omega$.

Therefore, we must consider an E.M.F. $e_2=-\mu\delta v=-\frac{\mu(x_1-x_2)}{\alpha_1}$ to be introduced into the plate circuit, μ being the magnification constant of the valve. The direction of the E.M.F. $\mu\delta v$ must be such as to increase the plate current when δv makes the grid more positive. At low frequencies, the capacity C_2 may be neglected, since it is shunted by the resistance R, and its value will only be a few micro-microfarads. Kirchoff's equations for the network may now be represented thus :—

$$
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
Z_1+\frac{1}{\alpha_1} & -\frac{1}{\alpha_1} & 0 & e_1 \\
-\frac{1+\mu}{\alpha_1} & \frac{1+\mu}{\alpha_1}+\frac{1}{\alpha_3}+R & -R & 0 \\
\frac{\mu}{\alpha_1} & -R-\frac{\mu}{\alpha_1} & R+Z_2 & 0
\end{array}
$$

Here α_3 is the conductance operator of the imperfect condenser C_3, i.e., $\alpha_3=G_3+j\omega C_3$.

Let D represent the operational determinant of this system of equations, and let D_{11} represent the minor of the element (1, 1), then

$$\frac{x_1}{e_1 D_{11}}=\frac{1}{D}$$

∴ The input impedance Z_g is given by

$$Z_g=\frac{e_1}{x_1}=\frac{D}{D_{11}} \qquad \text{with } Z_1=0.$$

This reduces to

$$Z_g=\frac{1}{a_1}\left[\frac{1-Wa_3}{1+Wa_3+\frac{a_3}{a_1}\left(1+\mu-\mu\frac{W}{Z_2}\right)}\right]$$

where $$W=\frac{RZ_2}{R+Z_2}$$

The input impedance will be measured as a capacity, and therefore it will be more convenient to consider the admittance

$$\frac{1}{Z_g}=\frac{a_1\left[1+Wa_3+\frac{a_3}{a_1}\left(1+\mu-\mu\frac{W}{Z_2}\right)\right]}{1+Wa_3} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

At telephonic frequencies a_3 is of the order 10^{-7}; and therefore since W is of the order 10^4, the term Wa_3 may be neglected in comparison with unity as a first approximation. Thus

$$\frac{1}{Z_g}=a_1\left[1+\frac{a_3}{a_1}\left(1+\mu-\mu\frac{W}{Z_2}\right)\right] \quad \text{approx.}$$

$$=a_1+a_3\left(1+\mu-\mu\frac{W}{Z_2}\right) \quad \text{approx.}$$

Substituting in this expression

$$a_1=G_1+jC_1\omega,$$

$$a_3=G_3+jC_3\omega,$$

and $$\frac{W}{Z_2}=\frac{R}{R+Z_2}=\frac{R}{R+R_2+jX_2}=\frac{R(R+R_2)}{(R+R_2)^2+X_2{}^2}-\frac{jX_2R}{(R+R_2)^2+X_2{}^2}$$

(where R_2 and X_2 are the external resistance and reactance in the plate circuit), we find

$$\frac{1}{Z_g}=G_1+G_3\left[1+\mu-\frac{\mu R(R+R_2)}{(R+R_2)^2+X_2{}^2}\right]-\frac{\mu RC_3\omega X_2}{(R+R_2)^2+X_2{}^2}$$

$$+j\omega\left\{C_1+C_3\left[1+\mu-\frac{\mu R(R+R_2)}{(R+R_2)^2+X_2{}^2}+\frac{\mu RC_3X_2\tan\delta_3}{(R+R_2)^2+X_2{}^2}\right]\right\}$$

Separating this into the capacity and conductance components, we see that the input capacity C_g is given by

$$C_g=C_1+C_3+\mu C_3-\frac{\mu RC_3[(R+R_2)-X_2\tan\delta_3]}{(R+R_2)^2+X_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The effective conductance is similarly given by

$$G_g=\left[G_1+G_3+\mu G_3-\frac{\mu RG_3(R+R_2)}{(R+R_2)^2+X_2^2}\right]-\frac{\mu RC_3\omega X_2}{(R+R_2)^2+X_2^2} \quad . \quad . \quad . \quad (3)$$

The phase angle φ_g, or the loss angle δ_g of the input impedance, may now be calculated from

$$\tan\delta_g=\cot\varphi_g=\frac{G_g}{C_g\omega}$$

The following points may be noted :—

1. When the load consists of a pure resistance, the formula for the input capacity becomes

$$C_g=C_1+C_3+\mu C_3-\frac{\mu RC_3}{R+R_2}=C_1+C_3+\frac{\mu R_2C_3}{R+R_2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This is the formula given by Miller for this case. The loss angle of the valve capacities does not appear in this expression, which explains the fact that Miller was able to verify the equation, although he neglected these power losses.

The conductance in this case is given by

$$G_g=G_1+G_3+\mu G_3-\frac{\mu RG_3}{R+R_2}=G_1+G_3+\frac{\mu R_2G_3}{R+R_2}$$

and, therefore, the phase angle is given by

$$\cot\varphi_g=\tan\delta_g=\frac{G_1+G_3+\mu R_2\dfrac{G_3}{R+R_2}}{\omega\left(C_1+C_3+\mu R_2\dfrac{C_3}{R+R_2}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Evidently, if $\tan\delta_1=\tan\delta_3$, then $\tan\delta_g=\tan\delta_1$, and does not vary with the load. In practice $\tan\delta_3$ is often less than $\tan\delta_1$, and consequently $\tan\delta_g$ decreases slightly as the load is increased.

2. When the load in the anode circuit is reactive, the input capacity is a function of this reactance, and is also affected by the dielectric losses associated with the grid-anode capacity ($\tan\delta_3$). Also when the reactance is positive (the load is inductive), the input conductance contains a negative term. This is often much larger than the positive term, which depends almost entirely on the dielectric losses in the valve as explained above. If these losses are such that $\tan\delta_1=\tan\delta_3$, then to a first approximation we may write the expression for the phase angle of the input circuit as

$$\cot\varphi_g=\tan\delta_g=\tan\delta_3-\frac{\mu RC_3X_2}{C_g[(R+R_2)^2+X_2^2]}$$

This equation, and also the capacity equation, have been verified by measurements made on an R valve at audio frequencies.

Experimental Investigation.

As a preliminary, the electrode capacities and power factors of the valve in its holder were determined. The Schering Bridge* was used for this purpose (*see* Fig. 2). The electrodes, whose mutual capacity was to be measured, were connected in parallel with the standard variable air condenser C_1. The third electrode was connected to earth. Balance was obtained with the valve so connected, by adjusting C_1 and C_4 (or C_3). The electrode which was connected to the point A of the bridge was then disconnected from the lead at X, and then the bridge re-balanced by varying C_1 and C_4 only. The difference between the two readings of C_1 gives the capacity between the two electrodes connected to A and B. (The capacities between these electrodes and the third electrode, which is earthed, do not affect the reading of C_1 appreciably, since one of these capacities is merely a shunt across the whole bridge, and the other is thrown in parallel with C_3, and thus only affects C_4.) The change in the reading of C_4 on disconnecting at X and re-balancing, is due entirely to the change in phase angle of the impedance AB on disconnecting the valve—i.e., it depends only

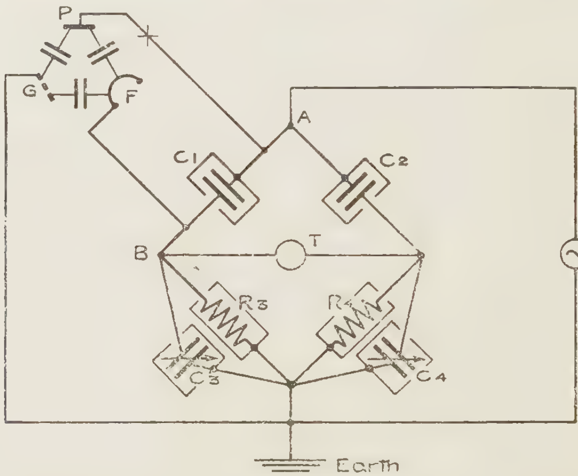

Fig. 2.—Bridge for the Measurement of Electrode Capacities.

on the phase angle of the valve impedance. (If disconnection were made on the B side of the arm AB, the value of C_4 would be affected, due to changes in earth capacities altering the value of C_3. Earth capacities at A being merely a shunt across the whole bridge, may be altered without affecting C_4.) If now C_4' is the reading of C_4 when the valve is connected to the bridge, and C_4'' when it is disconnected at X, then the power factor of the valve capacity being measured is given by

$$\tan \delta = \frac{C_1}{C_v} \omega \,.\, R_4 \,.\, (C_4' - C_4''),$$

where C_1=value of C_1 when valve is disconnected.

C_v=valve capacity, measured as explained above.

$\omega = 2\pi \times$frequency.

* *See* Hartshorn, Beama, Vol. 13, p. 92 (1923), or Proc. Phys. Soc., Vol. 36, p. 401 (1924).

In this way the capacities and power factors of the individual valve capacities were all measured.

The capacities and power factors of the valves may be also measured in pairs, in the manner adopted by Miller, and then the individual capacities and power factors obtained by calculation. The method here described is, however, much more direct, and also takes much less time.

An *R* valve was tested by both methods and the results were found to agree to within $0{\cdot}05\mu\mu$F. In general the uncertainties due to the effect of leads will be greater than this. The results obtained for the *R* valve used are given in Table I.

TABLE I.—*Capacity and Power Factor of Valve R*10 *in Holder.*
Frequency, 1,000∾ .

Electrodes.	Capacity $\mu\mu$F.	Tan δ.
Filament-Plate	$8{\cdot}_6$	0·24
Filament-Grid	$8{\cdot}_7$	0·20
Grid-Plate	$3{\cdot}_3$	0·13

The values of power factor are seen to be very considerable. Probably they are higher than the average for an *R* valve (cf., typical values given in a previous Paper, "Experimental Wireless," p. 263, Feb., 1925), but the fact that such power factors are not uncommon should be recognized.

These high power factors are probably due to absorption effects in the glass, and consequently they are accompanied by changes in the capacity with frequency. The results given in Table II illustrate this.

TABLE II.—*Change of Capacity with Frequency for Valve R*10.

Frequency.	Capacity $\mu\mu$F.	Power Factor.
500 ∾	$9{\cdot}_9$	$0{\cdot}2_0$
1,000	$8{\cdot}_7$	$0{\cdot}2_0$
2,000	$7{\cdot}_6$	$0{\cdot}2_0$
3,000	$7{\cdot}_0$	$0{\cdot}2_2$

The other capacities of this valve, and also those of other valves of the same make, behaved in much the same way.

The input impedance was also measured on the Schering Bridge, the connection being as in Fig. 3, which is self-explanatory. The impedance, which may be regarded as an imperfect condenser, was measured in exactly the same way as the valve capacities and power factors, disconnection being made at X (Fig. 3). The calculation of results is exactly the same as before. The case of negative power factor presents no difficulty. It merely means that, when the disconnection is made, the reading of C_4 increases instead of decreasing, and thus a negative sign appears when the usual formula is applied. The physical significance of the negative power factor is as follows. The input circuit behaves like a condenser in which the energy stored when the voltage is increasing is less than that given out when it is decreasing, i.e., there is a net gain of energy, this energy being, of course, received from the anode circuit by way of the grid-anode capacity. The current in the input circuit leads the potential difference between filament and grid, by an angle greater than 90 deg., or, alternatively, the filament-grid impedance contains a

negative resistance component. The valve does not, in general, oscillate since the complete input circuit includes the bridge, which has a positive resistance, in general, larger than the negative resistance introduced by the valve.

The alternating voltage applied to the grid in most of these measurements was of the order 0·3 volt. The detector T consisted of an amplifier and telephone. There was no difficulty in obtaining sensitivity to $0{\cdot}1\,\mu\mu$F. A grid bias of -2 or -4 volts was used, the anode voltage being usually of the order of 100.

Results.

(*a*) *Effect of Lighting the Filament on Valve Capacities.*

The input capacity and power factor of a valve at 1,000 ∾ were measured, first, with the filament current switched off, and then with various values of filament current up to the maximum. The circuit was as in Fig. 3, but there was no

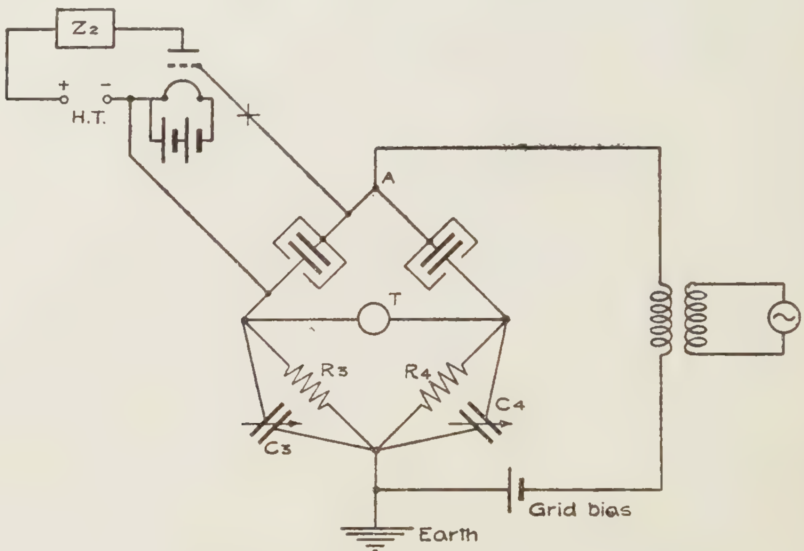

Fig. 3.—Bridge for Measurement of Input Impedance.

impedance Z_2 added in the anode circuit, except the resistance of the H.T. battery (about 100 ohms). The results are given in Table III.

Table III.—*Input Capacity and Filament Current for Valve R*10.
Frequency, 1,000 ∾. Anode Voltage, 60. Grid Bias, -2 volts.

Voltage Across Filament.	Input Capacity $\mu\mu$F.	Power Factor (tan δ).
0·0	$13{\cdot}_0$	0·21
2·7	$13{\cdot}_0$	0·22
3·2	$13{\cdot}_7$	0·22
4·0	$14{\cdot}_8$	0·38

It is to be noted that the capacity measured when the filament is not alight is $(C_{gf}+C_{ga})$ or (C_1+C_3). When the filament is alight, the capacity measured is given by Equation (4), where $R_2=100$ ohms, which is negligibly small, since R,

for a valve of this type, is about 30,000 ohms. Thus Equation (4) gives for the capacity $C_g = C_1 + C_3$, i.e., the capacity measured should not be altered by lighting the filament, unless the electrode capacities are affected by the presence of the electron current. Table III shows that the capacities are practically unchanged by lighting the filament. The small changes which are observed may well be due to the increased dielectric absorption effects which occur when the glass is heated. The increase in the power factor as the filament voltage is increased, supports this view. Similar measurements were made on a second valve of the same type. The results were much the same, except that the increase of power factor was even greater.

(*b*) *Variation of Input Impedance with Filament Voltage, Anode Voltage and Input Voltage.*

Before attempting to verify the various formulæ developed, it is advisable to know what fluctuations in input impedance are to be expected from changes in the experimental conditions. Table IV shows the effect of variations of filament voltage on the input impedance. It is evident that, when the valve is run at its rated voltage, small variations in this voltage do not seriously alter the input impedance.

TABLE IV.—*Input Impedance and Filament Voltage for R Valve.*
Anode Volts, 120. Grid Bias, −4 Volts. Load in Anode Circuit, Inductance Coil of 5 Henries. Frequency, 1,000 ∾.

Filament Voltage.	Input Capacity C_g	Tan δ_g.
4·0	$64{\cdot}_7 \mu\mu$F.	−0·50
3·9	$64{\cdot}_6$	−0·51
3·8	$62{\cdot}_3$	−0·55
3·7	$60{\cdot}_3$	−0·58

Variations of anode voltage were found to cause larger variations in the input impedance. Fig. 4 shows the results obtained with an *R* valve. In subsequent tests made with this valve the anode voltage was maintained at 120, since at this point, the changes of input impedance due to small changes of anode voltage are not large.

As previously stated, the majority of these measurements were made with a P.D. of 0·3 volt applied to the grid circuit of the valve. In order to find out whether the input impedance varied with the applied voltage, a series of measurements was made on one circuit, using various values of input voltage. The results are shown in Table V.

TABLE V.—*Input Impedance and Input Voltage for Valve R*10.
Anode Voltage, 120. Grid Bias, −4 Volts. Filament Voltage, 4. Load in Anode Circuit, Inductance Coil of 5 Henries. Frequency, 1,000 ∾.

Input Voltage.	Input Capacity $C_g \mu\mu$F.	Tan δ_g.
0·33	$66{\cdot}_7$	−0·49
0·16	$66{\cdot}_6$	−0·49
0·11	$66{\cdot}_5$	−0·49
0·03	67	−0·49

Equally constant results were obtained when the load in the anode circuit was a pure resistance. Within the experimental error the value of the input impedance is independent of the applied voltage over the range 0·03 to 0·3 volt. The bridge

and detecting arrangements used were not sufficiently sensitive to make reliable measurements with an applied P.D. less than 0·03 volt, but it seems probable that the input impedance can be regarded as completely independent of the applied voltage unless this is made very large.

(*c*) *Variation of Input Impedance with Load in the Anode Circuit.*

(A) *Pure Resistance Load.*—Standard wire-wound non-reactive coils of resistances 10,000 to 50,000 ohms were inserted in the anode circuit and measurements of input impedance made for each value used. In each case the H.T. battery voltage was adjusted until the P.D. between the anode and negative filament terminal was 120 volts approximately. The results are shown in Fig. 5. The input capacity

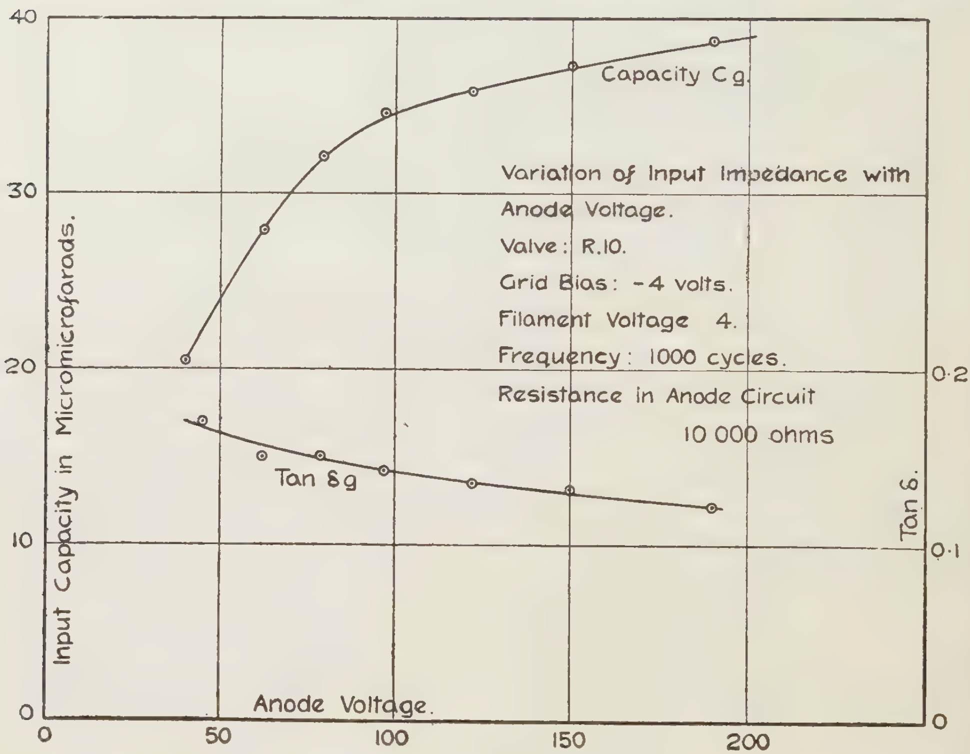

FIG. 4.

is seen to increase as the added resistance R_2 increases. This is in accordance with equation (4),

$$C_g = C_1 + C_3 + \frac{\mu R_2 C_3}{R + R_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The value of C_g when the added resistance is zero is $C_1 + C_3$, and thus this quantity is measured directly. The capacity increment $C_g - (C_1 + C_3)$ can therefore be readily obtained for each value of R_2. Equation (4) may be written in the form :—

$$\frac{1}{C_g - (C_1 + C_3)} = \frac{R}{\mu C_3} \cdot \frac{1}{R_2} + \frac{1}{\mu C_3}$$

Thus, if the reciprocal of $C_g-(C_1+C_3)$ is plotted against the reciprocal of R_2, the result should be a straight line. This was found to be the case, and from the slope of this line $\left(\frac{R}{\mu C_3}\right)$, and its intercept on the axis of ordinates $\left(\frac{1}{\mu C_3}\right)$ the following values of the valve constants were obtained :—

$$R=29{,}000 \text{ ohms.}$$

$$\mu C_3=63\mu\mu\text{F.}$$

This value of the valve resistance is in good agreement with the value deduced from its characteristic curves. The value of μ for this valve was found to be 8·3, which

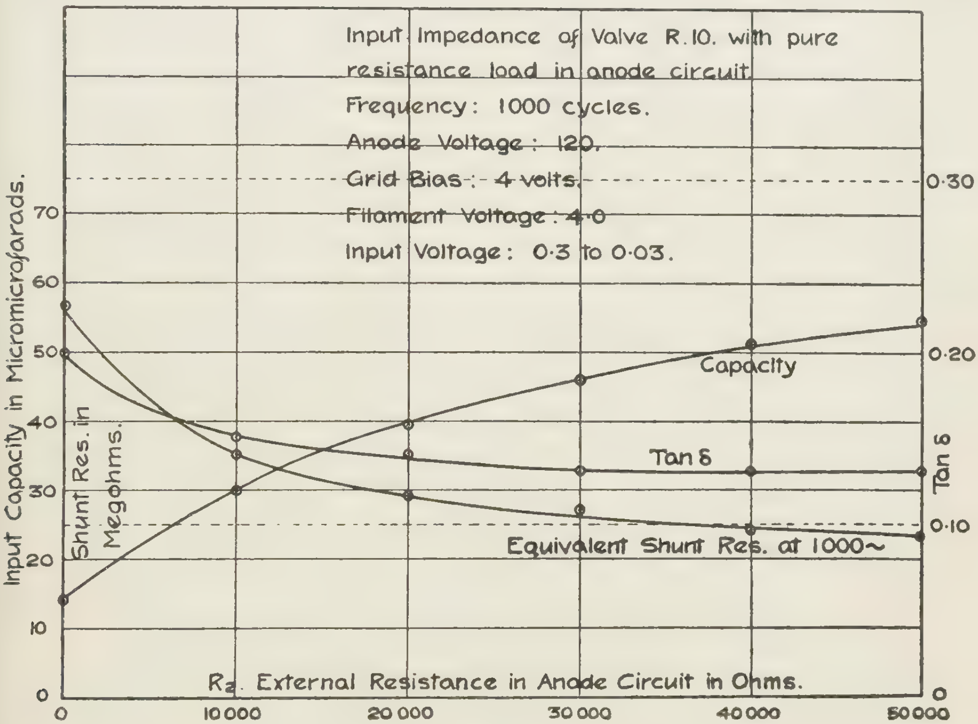

FIG. 5.

gives a value of $7{\cdot}_6\mu\mu$F. for the grid-anode capacity C_3. The value for C_3 obtained by direct measurement (Table I) was $3{\cdot}_3\mu\mu$F. This represents the value for the valve in its holder, excluding any connecting leads. Evidently the effective value of C_3 when the valve is connected in circuit with other apparatus is higher than this. In the present case there is an additional $4{\cdot}_3\mu\mu$F. which probably represents the capacity between the leads to the grid, and the leads to the anode, including the load Z_2 (Fig. 3). In Fig. 5 the observed points are marked with small circles. The capacity curve has been drawn through points calculated from equation (4), using the above values of the constants μC_3 and R. The agreement between the observed and calculated capacities is very close. Considering now the phase angle,

the value of tan δ was found to decrease from the value 0·20 when the added resistance R_2 was zero to a constant value of 0·13 for values of R_2 greater than 30,000 ohms. This is explained by equation (5). When R_2 is zero the value of tan δ_g is $\frac{G_1+G_3}{\omega(C_1+C_3)}$—i.e., it is the value for the two condensers C_1 and C_3 in parallel. This is evidently 0·20. If now R_2 is increased until it becomes comparable with R, tan δ_g approaches its limiting value $\frac{G_3}{\omega C_3}$ or tan δ_3, since in equation (5) the first term in numerator and denominator then becomes small compared with the second and third terms. Thus when R_2 exceeds 30,000 ohms the value of tan δ_g is 0·13, which is the value given in Table I for the tan δ of the grid plate capacity C_3. This agreement is partly accidental, since the values given in Table I exclude leads capacities. The existence of these would reduce the power factor, but the heating effect of the filament current increases the losses, and so brings the power factor back more or less to its previous value.

The curve for the equivalent shunt resistance at 1,000 ∿ is also shown in Fig. 5. This has been obtained by calculation from the values of C_g and tan δ_g. For this valve with a pure resistance load the equivalent shunt resistance at 1,000 ∿ is of the order of 30 megohms, and varies with the load in much the same way as tan δ. At higher frequencies the equivalent shunt resistance would, of course, be much smaller, since it is roughly inversely proportional to the frequency.

(B) *Load Purely Inductive.*—Standard self-inductance coils of various sizes from 0·1 to 31 henries were in turn inserted in the anode circuit, and measurements of input capacity and phase angle made for each case as before. The resistances of the coils were small compared with their reactances at 1,000 cycles per second, so that the resistance was neglected, and the input capacity and phase angle measured, considered merely as a function of the reactance added in the anode circuit. The capacity results are shown in Fig. 6 and those for tan δ_g in Fig. 7. The variation of capacity should be given by equation (2), viz.,

$$C_g=C_1+C_3+\mu C_3-\frac{\mu RC_3[(R+R_2)-X_2\tan\delta_2]}{(R+R_2)^2+X_2{}^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where X_2 represents the added reactance and R_2 the added resistance, which is there taken as negligible. When X_2 is zero the value of C_g given by the above equation is C_1+C_3. Thus this quantity can be measured directly. In the present case

$$C_1+C_3=15\mu\mu F.$$

When the added reactance X_2 is very large, we find from equation (2) that $C_g=C_1+C_3+\mu C_3$. Thus the input capacity C_g approaches this limit as the added reactance is increased indefinitely. In Fig. 6 the capacity is seen to approach a limiting value of $104\mu\mu F$. Hence

$$C_1+C_3+\mu C_3=104\mu\mu F.$$

and $$\therefore \quad \mu C_3=104-15=89\mu\mu F.$$

This value for μC_3 is higher than that obtained for a pure resistance load. This is doubtless due to a small increase in the component due to leads, etc., of the capacity C_3. The quantity C_1+C_3 is also correspondingly larger in this case.

Taking now the previously determined value of R—viz., 29,000 ohms—the values of C_g for various values of X_2 were calculated from (2). The value of tan δ_3 is 0·13 when the valve is cold (Table I). The value $0{\cdot}1_5$ was used in these calculations. The curve drawn in Fig. 6 represents the values of C_g obtained by calculation in this way. The points marked with small circles represent the observed values. The agreement is well within the experimental error, since slight changes

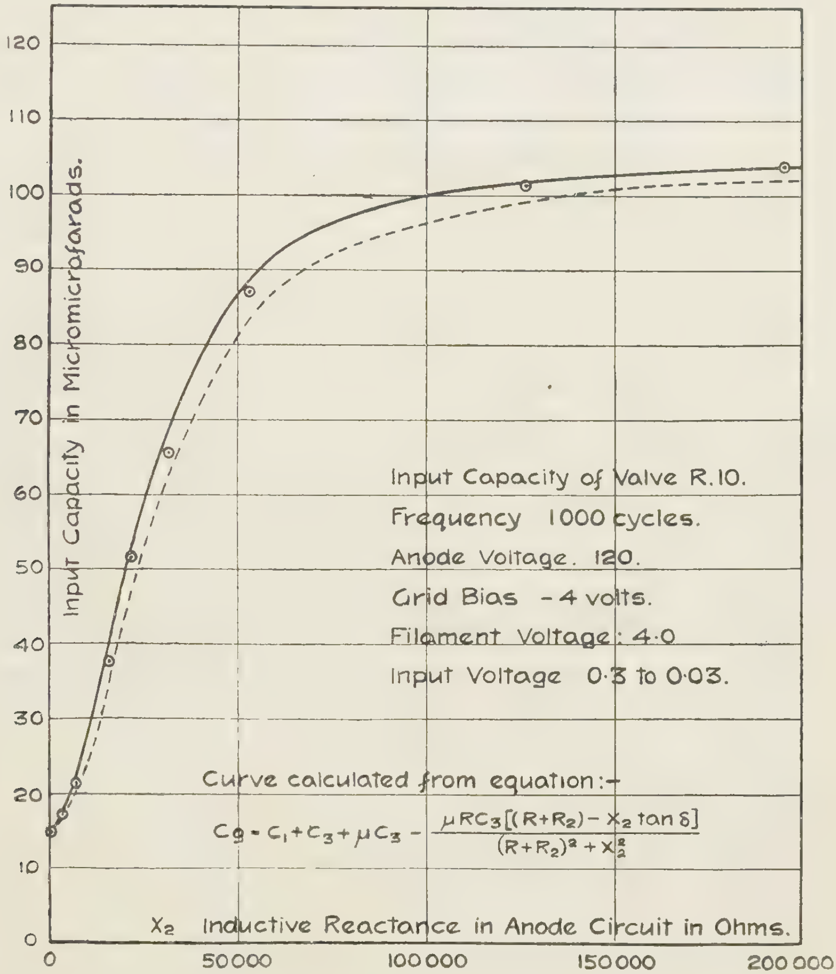

Fig. 6.

in C_3 are almost inevitable when the added inductance is changed—i.e., when a larger or smaller coil is added. The part played by power losses in the valve in increasing the input capacity is made clear by the dotted curve in Fig. 6. This represents the calculated values of C_g neglecting the term involving tan δ_3. The vertical distance between the dotted and continuous curves thus represents the capacity increment due to power losses.

Turning now to the phase angle φ_g of the input circuit, which is represented by tan δ_g [$=\cot \varphi_g$] in Fig. 7, we see that when the added reactance is zero the value of tan δ_g is 0·21—i.e., the input circuit behaves like a condenser with a power factor of approximately 20 per cent.—i.e., the current leads the voltage by an angle of 79 deg. As the added reactance is increased tan δ_g rapidly decreases, reaching a minimum value of $-0{\cdot}7_5$ when $X_2=12{,}000$ ohms (approx.). At this point the

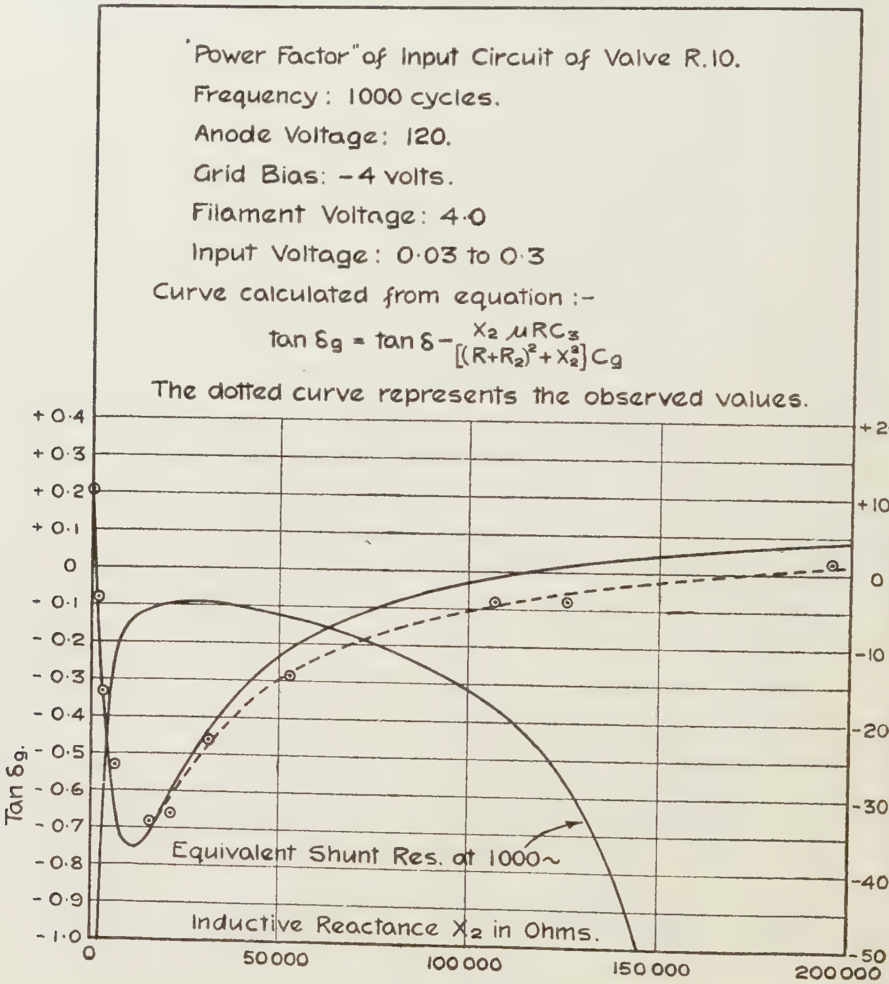

Fig. 7.

input circuit behaves like a condenser with a negative power factor (sin δ_g) of 60 per cent.—i.e., the current in the grid circuit leads the grid-filament P.D. by 126 deg.—since $\tan^{-1}(-0{\cdot}7_5)=\sin^{-1}(-0{\cdot}60)=36$ deg. As the value of X_2 is increased still further the value of tan δ_g increases, becoming positive again for very large added reactances. The full expression for tan δ_g obtained from (2) and (3) is rather cumbersome, and in order to simplify it, a further approximation has been made. It

is evident from (3) that the full expression for tan δ_g will contain two terms, the first being a function of the power losses in the valve, and the second being a function of the added reactance. In general, the first term will be the smaller, and it varies comparatively little with the added load. It has therefore been assumed constant, and has been denoted by tan δ. We then have

$$\tan \delta_g = \tan \delta - \frac{X_2 \mu R C_3}{[(R+R_2)^2+X_2{}^2]C_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

When $X_2=0$ is inserted in this equation we have $\tan \delta_g = \tan \delta$, and therefore tan δ has been taken as 0·21, which is the measured value of tan δ_g when $X_2=0$. Since

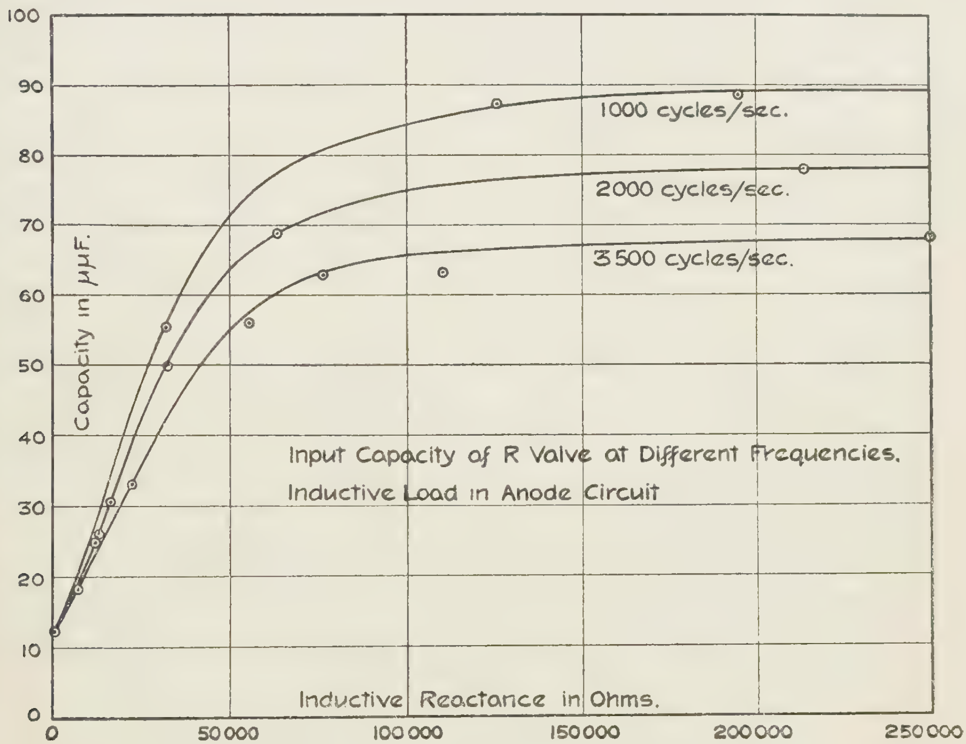

FIG. 8.

all the other quantities in (6) are known, the curve showing the relation between tan δ_g and X_2 can be plotted. This is the continuous curve in Fig. 7. The observed points lie on the dotted curve. The agreement between the two is as close as can be expected. The divergence is evidently due to the fact that the term tan δ is not strictly speaking constant. As a matter of fact, the value of tan δ will decrease with added load, in much the same way as it does in Fig. 5, as may be seen by a comparison of the equations for the two cases. The calculated value for tan δ_g will therefore be slightly high for large added reactances, which explains the divergence of the two curves. We may therefore conclude that the equations given, represent

the input impedances of valves at low frequencies with sufficient accuracy for all practical purposes.

In this case, also, the equivalent shunt resistance curve at 1,000 ∿ has been calculated from those of capacity and tan δ. It is shown in Fig. 7. The smallest value of the equivalent negative shunt resistance is 5 megohms at 1,000 ∿. This quantity is, of course, enormously dependent on the frequency—e.g., at 10,000 cycles it would be one-tenth of this amount if the value of tan δ were unchanged—and to a first approximation tan δ is independent of frequency for a given added reactance.

(C) *Variation of Input Impedance with Frequency.*—The present investigation applies only to low frequencies (the telephonic range, say), and for this case the equations indicate that for a given added impedance in the anode circuit the input impedance is independent of frequency, provided that the valve constants μ, R, C_3, C_1 are independent of frequency. For the valve used in these experiments the values of C_1 and C_3 were found to vary with frequency (*see* Table II). Exactly corresponding variations were found in the input impedance. Fig. 8 shows curves giving the variation of input capacity with added reactance in the anode circuit for a number of different frequencies. From the curves we may deduce the values of the valve constants C_1+C_3, μC_3, and therefore C_3, in the manner previously indicated. The following results were obtained in this way :—

Frequency.	C_1+C_3	μC_3	C_3
1,000	$12 \cdot_5$	76	$9 \cdot_2$
2,000	$12 \cdot_0$	66	$8 \cdot_0$
3,500	$11 \cdot_3$	58	$7 \cdot_0$

The changes are of the same order as those shown in Table II. The curves in Fig. 8 were calculated from the formula (2), using these values of C_1+C_3 and μC_3 and the same values of R. The agreement is sufficient to show that the change in input capacity is due solely to the change in valve capacities. Similar results were obtained for a pure resistance load in the anode circuit.

In carrying out this investigation, I have had the privilege of discussing various points with a number of my colleagues, and I wish to thank them for valuable suggestions, particularly Messrs. D. W. Dye, T. I. Jones, and F. M. Colebrook. My thanks are also due to Mr. D. A. Oliver for help in making the observations.

DISCUSSION.

Mr. J. Nicol said that the Paper had interested him very greatly. Some of the details mentioned by the author in presenting it to the meeting might advantageously be added to the Paper for publication in the Proceedings ; for instance, the diagram of the Schering Bridge or a reference to a published description of this, and the method of increasing the sensitivity of the bridge by using a two-valve amplifier in the detector arm. Would not the mathematical expressions be simplified if admittances were used throughout instead of impedances ?

Prof. C. L. Fortescue said that it was a new and gratifying experience to meet with valve-impedance measurements which agree with theory. The Paper had an important bearing on a subject not mentioned in it—namely, the behaviour of valve voltmeters. It now appeared that variations in the load impedance of the valve had important effects on the effective input conductance, and might render this negative, so that the reaction on the measured circuit might be important. Fig. 7 of the Paper bore out the contention that the behaviour of the valve voltmeter is by no means simple. In amplifiers the input conductance was of greater practical importance than the input capacity, which was small, and usually swamped by larger shunt

capacities. Thus equation (3) of the Paper, not equation (2), was of prime importance in connection with the howling which arises from negative resistance. The theory in the Paper was presented in the American manner, the ouput voltage variations being expressed as a fraction of the input variations ; possibly the alternative method, in which the parameters of the valve are employed, would give the same results in a simpler and more direct manner. In conclusion, he asked for information as to the lower limit of the voltage range of which the bridge could take account.

Mr. T. G. Hodgkinson : I should like to join in congratulating Mr. Hartshorn on the careful measurement of some of the quantities we talk about so flippantly in these days. It is interesting to attack the problem from another point of view—that is, the point of view of the design of capacity coupled oscillators for the shorter waves. For oscillations below, say, 350 metres, with quite ordinary coil dimensions, the capacity between the electrodes (grid and anode) is sufficient to maintain the system in oscillation, and the part this capacity plays in both the determination of the oscillation frequency and the power output is important. For example, a valve system with inductances in the grid and anode circuits, not coupled magnetically, has at least two free periods, a longer period and a shorter period, which can be predetermined in terms of the inductances, resistances, capacities and inter-electrode capacity of the combination. The longer period (I call it the sum mode) is supported in oscillation by the grid-anode capacity, and I can confirm the order of the author's results from cases of this kind which I have predetermined and measured.

Dr. E. H. Rayner said that he thought the author should put on record the terminal points across which his measurements were taken. Did he include in the valve impedance the wire leads, the holder and the cap of the valve ? The insulating composition in the cap, if not removed for the purpose of the experiments, would account for the power factor to a large extent.

Mr. J. Nicol (subsequent contribution) : With reference to Prof. Fortescue's remarks on errors introduced by valve voltmeters, I believe that in the Moullin voltmeter a large condenser connects the plate and filament legs together, bridging the microammeter—there is no H.T. battery. Though the author does not consider the case of a capacitative loading, a reference to his curves will show that when the load impedance is very low the input impedance of the valve will be simply that of the grid filament condenser, and therefore very high, so that the voltmeter should in most cases have very little effect on the voltage it is used to measure. Of course, if the bridging condenser is omitted great errors may easily be introduced, as Prof. Fortescue suggests.

Author's reply : Replying to Mr. Nicol, the two-stage transformer-coupled amplifier used in the detector arm was not specially designed for this work, and presents no novel features. Almost any amplifier will do. Convenient values for the resistance arms of the bridge are 1,000 or 5,000 ohms. The variable air condensers had maximum values ot 0·001 or 0·002 microfarad. Impedances were used for the parts of the circuit external to the valve, since in practice the unit of impedance, the ohm, is much more convenient to handle than the unit of admittance.

I quite agree with Prof. Fortescue as to the importance of the input conductance. I have no information as to the minimum input voltage required to give sufficient sensitivity for these bridge measurements. It will, of course, depend largely on the amplifier used as detector and the degree of freedom from stray disturbances. I have little doubt that an amplifier could be designed which would give sufficient sensitivity with input voltages considerably less than 0·03, the smallest used by me in these experiments.

I am very interested to hear that Mr. Hodgkinson can confirm the order of my capacity results from observations on capacity coupled oscillators.

In reply to Dr. Rayner, I may say that all the measurements were made with the valve mounted in its holder, so that the effects of capacity and power loss in the leads, holder and cap of the valve are included in the impedances measured.

IX.—A PRINCIPLE GOVERNING THE DISTRIBUTION OF CURRENT IN SYSTEMS OF LINEAR CONDUCTORS.*

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Abstract.

A brief résumé is given of the procedures which have been developed for determining the distribution of direct current in systems of linear conductors. In this connection reference is made to practically all the laws, theorems, principles and procedures generally considered to pertain to this particular field of investigation. Consideration is then given to a principle which when employed usually leads more directly to the solution of problems than does any of the procedures commonly used.

This principle applies to systems of linear conductors in which the currents are proportional to the impressed electromotive forces ; the electromotive forces may be any function of time, and may be distributed in any manner throughout the system ; and the branches may contain resistance, inductance, capacitance or any two or all of these in series, may be so arranged as to move with respect to a permanent magnet, thus developing counter electromotive forces, and may be connected by contacts or mutual inductances or both of these. For such a system of conductors the current in any branch is that which would result if all impressed electromotive forces were replaced by a single impressed electromotive force, located in the particular branch and equal to the drop in potential which originally would have appeared across the break had this branch been opened. While this principle is a logical consequence of well-known laws, it has been used but very little and seems to be practically unknown. It is shown here that it may be used to advantage in all or practically all cases in which the conductors form a series-parallel combination or a network which may be changed to a series-parallel combination by opening the branch in which it is desired to determine the current.

1. Introduction.

In electrical measurements and the distribution of electric power it is often necessary to estimate or determine the current to be expected in one or more branches of a system of conductors in advance of assembly or construction, or when conditions are such that a direct measurement is not practicable. A case in point is the determination of the current in the galvanometer branch of a Wheatstone bridge network for a given set of conditions.

In the discussion of this subject it will be presumed, at least for the greater part, that the currents are direct and constant, so that currents and electromotive forces may be added algebraically, and the resistances only of the branches need be considered. However, it should be understood that the same procedures may be followed when the currents are alternating, and also when the currents are transient, provided due consideration is given to those electromotive forces resulting from changes of the current in branches containing inductance or capacitance, motion of branches free to move with respect to the field of a magnet, etc.

A straightforward and obvious procedure for the solution of the problem stated above, and similar problems, originally proposed by Kirchhoff,[1] is to apply Ohm's law, or Kirchhoff's second law, to $n-m+1$ of the enclosed meshes or circuits of the

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system, and Kirchhoff's first law[2] to $m-1$ of the branch points, where m is the number of branch points and n the number of branches. This procedure gives n simultaneous equations from which the current in each of the n branches may be determined in terms of the electromotive force of the battery and the resistances. However, since for the simplest bridge n is 6, it will be seen that for many cases the algebra involved in the solution of the n simultaneous equations will be long and tedious. Therefore attempts have been made to develop less complicated procedures, and these have led to a marked improvement in the situation.

By the use of mesh or cyclic currents, which is a special application of the principle of superposition,[3] instead of Kirchhoff's first law, Maxwell[4] has shown that a partial solution may be obtained from $n-m+1$ simultaneous equations, and that a complete solution may be obtained simply by adding expressions given by the partial solution. A very thorough discussion of the subject, based mainly on the contributions made by Kirchhoff and Maxwell is given by Feussner.[5]

Further, it has been noted that in some cases a knowledge of the current in one branch in terms of the current in another branch and the resistances will serve as well as a knowledge of the current in terms of the electromotive force of the battery and the resistances. In such cases the number of unknowns is $n-1$, and consequently a partial solution may be obtained from $n-m$ simultaneous equations.[6] Therefore a partial solution for the Wheatstone bridge network may be obtained from two simultaneous equations, whereas the procedure proposed by Kirchhoff requires six. A discussion of the subject, including the contributions made by Kirchhoff, Maxwell and Callender, and extended so as to apply when the current is alternating, is given by Hague.[7]

In still other cases it is sufficient if the currents are known only approximately, or when some special relation exists between the resistances. In many such cases it is possible to avoid entirely the use of simultaneous equations. For example, in a particular case it may be obvious that the current in one branch is small in comparison with the currents in other branches, and that if this branch were removed the system would be changed from a network to a group of series-parallel conductors. The current in all branches except the one may then be calculated as though this one were not a part of the system, and the values thus obtained will not be materially in error. In some cases, too, it is possible to determine to a close approximation the current in that branch in which the current is small in comparison with the currents in other branches. Illustrations of this are the determination of the sensitivity of the Wheatstone and other bridges, potentiometers, etc., by Smith,[8] and the determination of the sensitivity of the Thomson bridge by Northrup.[9] In these determinations, both considered the current through the galvanometer branch when the bridge is very nearly balanced to be that which would result from an electromotive force in one of the arms equal to the current in this arm times the change in resistance in this arm necessary to reduce the current in the galvanometer branch to zero. A proof of this principle, given by Smith, is based on Kirchhoff's reciprocal theorem,[10] a theorem which may be used to advantage in the solution of many problems in current distribution.

Another theorem of importance in this connection pertains to the equivalence of stars and delta in systems of linear conductors. Kennelly[11] has shown that any three-point star (three-way branch-point) may be replaced by a delta or triangle, and, conversely, any delta may be replaced by a three-point star. Further, Rosen[12]

has shown the equivalence of any n-point star and a delta or network in which there are $1/2\ n(n-1)$ conductors connecting each of the n points to every other point. The use of this theorem leads to material simplification in the calculation of the resistance or impedance between any two points of a network. The theorem also has other applications.

In this brief résumé of the subject reference has been made to most of the laws, principles, theorems and procedures which pertain to the distribution of direct currents in systems of linear conductors.

There is, however, another principle which in some cases is of material assistance in the solution of problems of this type. This principle may have been applied to direct current bridges(13) by Jaeger, Lindeck and Diesselhorst. It has been applied to alternating-current bridges(14) and to other problems(15) by myself, and with direct currents has been applied to a general network(16) by Harrison and Foote. However, except as applied to simple circuits, it seems to be practically unknown. The purpose of this Paper is to state this principle, to prove its validity, to show to what types of problems it is applicable, and to show the advantages to be gained by its use.

II. Statement and Proof.

Let it be assumed that the system of conductors is one in which

1. Each branch is linear.
2. Ohm's law is applicable to each branch.
3. Impressed electromotive force may be distributed in any manner throughout the system.

For such a system of conductors, or that described in an appendix to this Paper, *the current in any branch is that which would result should an electromotive force, equal to the potential difference which would appear across the break were the branch opened, be introduced into the branch and all other electromotive forces be removed.*

This principle may be developed from well-known laws and principles as follows :—

1. From Ohm's law and the principle of superposition it follows that each electromotive force causes a current in each branch proportional to itself and independent of the currents caused by other electromotive forces, and the current in any branch is the algebraic sum of the currents in that branch caused by the various electromotive forces.

2. From 1 it follows that if in any branch there is introduced an electromotive force of the proper sign and magnitude to reduce the current in this branch to zero, the current in this branch caused by this electromotive force is equal in magnitude but opposite in sign to the current in this branch caused by the other electromotive forces in the system.

3. With the current in any branch equal to zero, the branch may be opened without changing the current in any branch or the potential difference between any two points in the system. Consequently, for the conditions given in 2, opening the branch causes no potential difference to appear across the break.

4. With the branch open, any or all of the electromotive forces in it may be removed without changing the current in any branch of the system ; but with the

appearance of a potential difference across the break equal in magnitude but opposite in sign to the electromotive forces removed.

5. Consequently, when a branch is opened, the potential difference which appears across the break is equal in magnitude but opposite in sign to the electromotive force which, if placed in this branch, would reduce the current to zero.

6. Therefore, if an electromotive force equal to the potential difference which would appear across the break should a branch be opened, were introduced into the branch under consideration and all other electromotive forces regardless of their location removed, leaving all branches closed, the current in it would be the same as that which is caused by the other electromotive forces.

As a result of discussions with my colleagues, independent proofs of this principle have been developed by Dr. A. S. McAllister and Dr. Chester Snow. These unpublished proofs differ from each other, from that given by Harrison and Foote,[16] and from that given above. Additional proof is contained in the solutions of a number of the problems which are considered later.

From the principle as stated and the principle of superposition it follows that—

1. For systems of conductors such as those described above or in the appendix, and in which there may be any number of electromotive forces, the change in current in any branch brought about by the introduction of an additional electromotive force is equal to the change in current which would be produced by the introduction into the particular branch of an electromotive force equal to the change which would occur in the potential difference across the break if such a break were present at the time the additional electromotive force is introduced.

2. For systems of conductors such as are described above, or in the appendix, the change in the current in any branch caused by the addition of a branch containing no electromotive force is equal to the change in current which would be produced in the particular branch by an electromotive force in the added branch equal to the potential difference which existed between the points to which the branch is connected, before the connection was made.

3. For systems of conductors such as those described above or in the appendix, except that in one branch the current is not proportional to the impressed electromotive force, the current in this branch is that which would result from an electromotive force, located in it, equal to the potential difference which would appear across the break were the branch opened.

III. Applications and Advantages.

To show something of the conditions under which the principle is applicable and the advantages to be gained by its use, it will be applied in the solution of a number of problems. As these are to serve as illustrations only, no effort will be made to get the solutions into a convenient form for use, or to explain their significance.

Problem 1.—To measure the potential difference V between two terminals of a system of conductors by means of a voltmeter when conditions are such that the connection of the voltmeter causes an appreciable lowering of the potential difference. One of two procedures for making such a measurement, proposed by Brooks,[17] is (*a*) connect a voltmeter to the terminals and note the reading V_1; (*b*) insert in series

with the voltmeter a resistance equal to the resistance of the voltmeter and note the reading V_2; (c) calculate V from the equation

$$V=V_1V_2/(V_1-V_2).$$

To show that this procedure gives a correct result, let

i_1 be the current through the voltmeter when the reading is V_1,

i_2 be the current through the voltmeter when the reading is V_2,

r be the resistance of the voltmeter, and

R be the resistance of the system of conductors between the terminals, between which the potential difference is to be determined.

According to the principle under consideration

$$i_1=V/(R+r) \text{ and } i_2=V/(R+2r),$$

while, presumably, the instrument is so graduated that

$$i_1=V_1/r \text{ and } i_2=V_2/r.$$

Therefore

$$V=V_1(1+R/r)=V_2(2+R/r),$$

from which it follows on the elimination of R/r that

$$V=V_1V_2/(V_1-V_2).$$

Problem 2.—One of two two-wire feeders supplying power from the same source to adjacent territories is found to be overloaded, and it is desired to know what shift

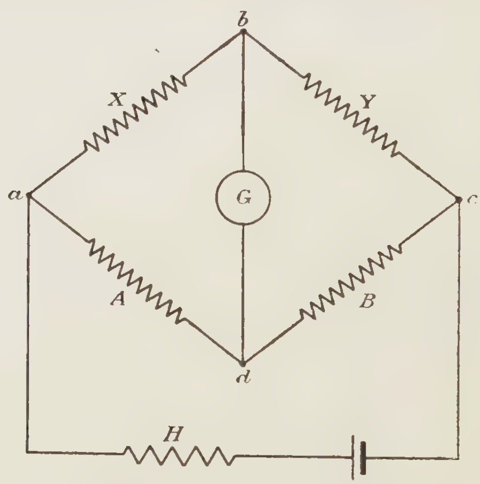

FIG. 1.—WHEATSTONE BRIDGE.

in load might be expected were a two-wire tie to be made between these. The conditions of the problem are as follows :—

(a) Each feeder is transmitting power nominally at 13,000 volts.

(b) At what seems to be a favourable place for making the tie, the voltage of the excessively loaded feeder, found by measurement, is 260 volts less than that of the other feeder.

(*c*) The resistance (or impedance) to an electromotive force located in the tie, estimated from the size and length of the conductors proposed for the tie and the size and length of the feeders is 3·25 ohms.

Therefore, according to the principle under consideration, making the tie would cause a shift of 260/3·25, or 80 amperes from one feeder to the other, and would relieve the excessively loaded feeder to the extent of $80 \times 13{,}000$ volt-amperes, or 1,040 k.v.a.

As a simple independent analytical method for solving this problem was not evident, the principle was applied to other problems of the same type and the results obtained checked experimentally.

Problem 3.—To determine the current through the galvanometer branch of a Wheatstone bridge network. Referring to Fig. 1, let

E be the potential drop from a to c with the galvanometer branch open.

X, Y, A and B be the resistance of the arms of the bridge.

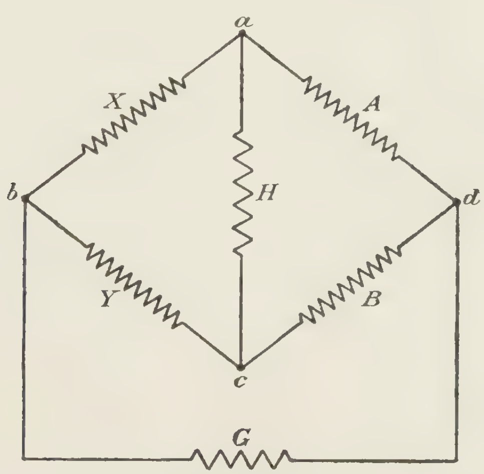

FIG. 2.—WHEATSTONE BRIDGE ARRANGED FOR SHOWING RESISTANCE OF COMPLETE GALVANOMETER CIRCUIT.

G be the resistance of the galvanometer branch.

H be the resistance of the battery branch.

R be the resistance of the galvanometer circuit—that is, the resistance to an electromotive force in the galvanometer branch.

i_g be the current in the galvanometer branch.

With the galvanometer branch open it may be seen by inspection that the potential drop from a to b is

$$EX/(X+Y),$$

and from a to d is

$$EA/(A+B)$$

The potential difference across the break in the galvanometer branch then is

$$EX/(X+Y)-EA/(A+B) \text{ or } E(XB-YA)/(X+Y)(A+B).$$

Therefore, if the galvanometer branch is closed

$$i_g = E(XB - YA)/(X+Y)(A+B)R.$$

If conditions are such that the drop in potential from a to c is not appreciably lowered by closing the galvanometer branch, it may be seen by inspection of Fig. 1 that

$$R = G + XY/(X+Y) + AB/(A+B) \text{ approximately.}$$

This expression for R applies either in case the bridge is approximately balanced or in case the resistance of the battery branch is very small in comparison with the resistance of the galvanometer circuit.

If an exact expression for R is desired, it may be obtained readily by transforming one of the deltas to a star, as suggested by Kennelly.(11) Replacing the delta bac of Fig. 2 (which is the same as Fig. 1, except that the battery and galvanometer

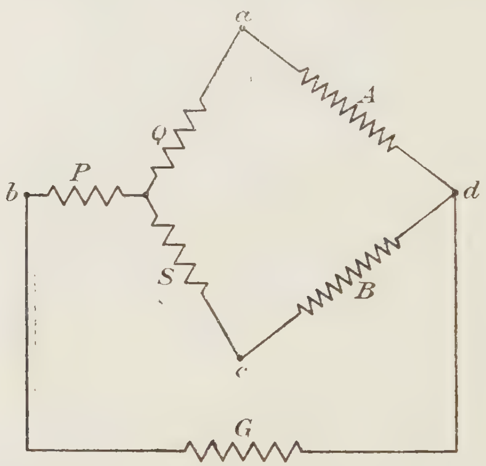

FIG. 3.—WHEATSTONE BRIDGE WITH THE DELTA bac OF FIG. 2 REPLACED BY THE EQUIVALENT STAR.

are considered to be replaced by conductors having equivalent resistances) by a star gives the arrangement shown in Fig. 3. Here

$$P = \frac{XY}{X+Y+H}, \qquad Q = \frac{HX}{X+Y+H}, \qquad \text{and } S = \frac{HY}{X+Y+H}$$

Therefore it may be seen by inspection of Fig. 3 that

$$R = G + P + (Q+A)(S+B)/(Q+A+S+B)$$

or

$$R = G + \frac{XY}{X+Y+H} + \frac{\left(\frac{HX}{X+Y+H} + A\right)\left(\frac{HY}{X+Y+H} + B\right)}{\frac{HX}{X+Y+H} + A + \frac{HY}{X+Y+H} + B}$$

Problem 4.—To determine the current through the galvanometer branch of the Brooks model 5 potentiometer (18) for which the circuit arrangement is that shown in Fig. 4.

To solve this problem, consider first that the galvanometer branch and the r_1 branch are open. Then the current in the r_6 branch will be

$$\frac{e_1}{r_3+r_6}$$

and the potential difference across the break in the r_1 branch will be this current times the resistance r_6, or

$$\frac{e_1 r_6}{r_3+r_6}$$

Now consider the r_1 branch closed, then the current in it will be this potential

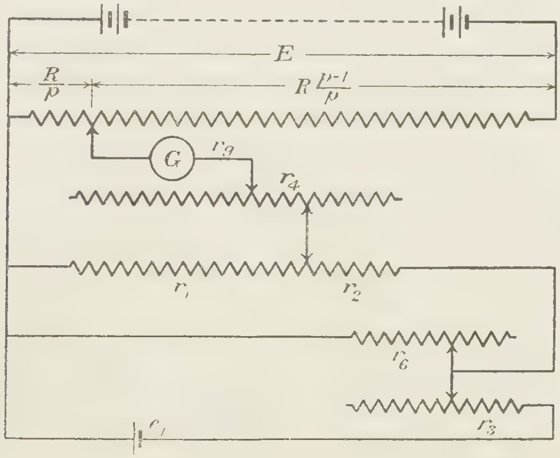

FIG. 4.—CIRCUITS OF BROOKS MODEL 5 DEFLECTION POTENTIOMETER.

difference considered as an electromotive force divided by the resistance to an electromotive force in the r_1 branch, or

$$\frac{\dfrac{e_1 r_6}{r_3+r_6}}{r_1+r_2+r_3 r_6/(r_3+r_6)}$$

The potential difference across the break in the galvanometer branch caused by the electromotive force e_1 then will be this current times r_1, and that caused by the electromotive force E, which is of opposite sign, is the current E/R times the resistance R/p. The potential difference across the break in the galvanometer branch therefore will be

$$\frac{\dfrac{e_1 r_6 r_1}{r_3+r_6}}{r_1+r_2+r_3 r_6/(r_3+r_6)} - \frac{ER}{Rp}$$

With the galvanometer branch closed, the current in the galvanometer branch will

be this potential difference considered as an electromotive force divided by the resistance of the galvanometer circuit, or

$$i_g=\frac{\dfrac{\dfrac{e_1 r_6 r_1}{r_3+r_6}}{r_1+r_2+r_3r_6/(r_3+r_6)}-\dfrac{ER}{Rp}}{r_g+r_4+\dfrac{r_1[r_2+r_3r_6/(r_3+r_6)]}{r_1+r_2+r_3r_6/(r_3+r_6)}+\dfrac{R/p\times R(p-1)/p}{R/p+R(p-1)/p}}$$

In the development of this equation terms once written down have not been changed even in those cases in which simplifications might readily have been made. Consequently it might have been written simply from an inspection of the diagram of connections, and, in slightly different form, was so written on first consideration of the problem. The equation may readily be reduced to the simpler form given by Brooks.

Problem 5.—To determine the values for the resistances of a shunt box for a galvanometer to satisfy the following conditions: (*a*) The current drawn from the

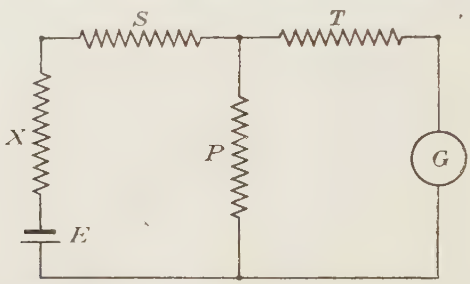

FIG. 5.—SHUNT FOR REDUCING SENSITIVITY OF GALVANOMETER WHILE KEEPING THE DAMPING AND CURRENT DRAWN FROM SOURCE CONSTANT.

source of the electromotive force after the deflection has reached a constant value shall be the same as though no shunt were used. (*b*) The resistance to the electromotive force developed by the motion of the coil of the galvanometer shall be the same as though no shunt were used. (*c*) The current through the galvanometer after the deflection has reached a constant value shall be $1/n$ times what it would be if no shunt were used.

Volkmann[19] has shown that these conditions may be fulfilled by the arrangement shown in Fig. 5, in which X represents the resistance and E the electromotive force of the source of the current, G the resistance of the galvanometer, and S, P and T the resistances of the added conductors. Then if I is the current drawn from the source and i_g is the current through the galvanometer, it follows from condition (*a*) that

$$I=\frac{E}{X+S+P(T+G)/(P+T+G)}=\frac{E}{X+G} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Therefore

$$S+P(T+G)/(P+T+G)=G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From condition (c) it follows that

$$i_g=\frac{E}{X+S+P(T+G)/(P+T+G)}\times\frac{P}{P+T+G}=\frac{E}{n(X+G)} \quad . \quad . \quad . \quad . \quad (3)$$

and dividing equation (3) by equation (1) gives

$$P/(P+T+G)=1/n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From the principle under consideration and condition (b) it follows that

$$i_g=EP/(P+S+X)(X+G)$$

and substituting the expression for i_g given by equation (3) gives

$$P/(P+X+S)=1/n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From equations (4) and (5) it follows that

$$T=S+X-G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and from equation (5) it follows that

$$P=(S+X)/(n-1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

These expressions for T and P substituted in equation (2) give

$$S=(Gn-X)/(n+1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This expression for S substituted in equation (5) gives

$$P=n(X+G)/(n^2-1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and this expression for P substituted in equation (4) gives

$$T=(nX-G)/(n+1).$$

These expressions for S, P and T are the same as those given by Edler,[20] who showed that the expressions given by Volkmann are unnecessarily complicated.

Problem 6.—To design a multiplier for a deflection instrument so that it may

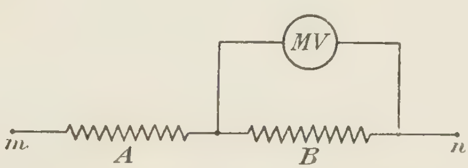

FIG. 6.—MULTIPLIER FOR MILLIVOLTMETER.

be used as a voltmeter with a range of 1·5 volts, when the only information available concerning its constants is that when 20 ohms is placed in series the reading is proportional to the applied potential difference and a full scale deflection is obtained for an applied potential difference of 0·15 volt. The problem, therefore, is to devise a system of conductors with the instrument connected into one branch, such that the current in this branch for any potential difference applied between a particular pair of terminals will be the same as for one-tenth this potential difference applied to the instrument with 20 ohms in series. A possible arrangement is that shown in Fig. 6, in which values for the resistances A and B are so chosen that with the instrument branch open the potential difference across the break is one-tenth the potential drop from m to n, and the resistance as measured from the position of

the instrument with the instrument removed and the terminals m and n connected by a conductor of negligible resistance is 20 ohms. This gives

$$B/(A+B)=0{\cdot}1 \text{ and } AB/(A+B)=20$$

or

$$A=200{\cdot}0 \text{ and } B=22{\cdot}22.$$

Here the potential difference between m and n has been considered as though it were produced by a source of electromotive force having no resistance. However, as it is the potential difference, and not the electromotive force, which produces it that is to be measured, the resistance or impedance of the source of the electromotive force does not enter into the problem.

In this case, if the potential difference is alternating, the impedance of the instrument may be presumed to increase with the deflection; consequently the current will not be proportional to the electromotive force. However, no limit need be stated or implied as to how nearly the current must be proportional to the

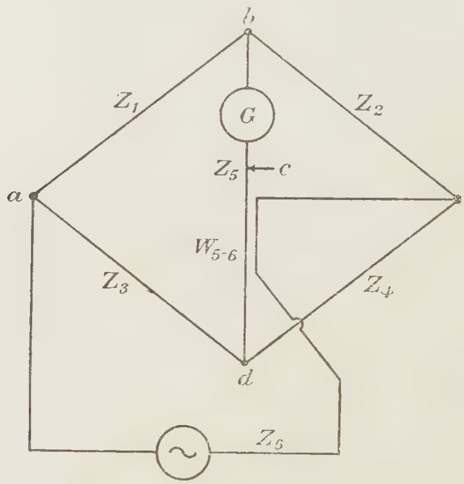

FIG. 7.—HUGHES BALANCE.

applied potential difference, provided only that any increase in the potential difference causes an increase in the current.

To show the applicability of the principle under consideration to somewhat more complicated problems it will be assumed that the currents are alternating. Here the symbolic notation will be used—that is, $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, etc., will be referred to as currents; $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, etc., will be referred to as electromotive forces or potential drops; $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, etc., will be referred to as impedances; and $\mathbf{W}_{1\text{-}2}$, $\mathbf{W}_{1\text{-}3}$, $\mathbf{W}_{2\text{-}3}$, etc., will be referred to as conjunctances; but it should be understood that each is a complex quantity, or what in alternating-current theory is often called a vector.[21] Further, it should be understood that $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, etc., each includes the "motional impedance,"[22] if present, as well as the electrical impedance, and $\mathbf{W}_{1\text{-}2}$. $\mathbf{W}_{1\text{-}3}$, $\mathbf{W}_{2\text{-}3}$, etc., each includes the effects of all types of coupling between the branches to which it pertains.

The solution of the problems will be left in the complex form, since to rationalize

them would add a considerable amount of detail which is not required for the purpose at hand.

Problem 7.—To determine the current through the detector branch of the Hughes' balance[23] for which the circuit arrangement is that shown in Fig. 7. It is to be understood that the detector branch 5 and the generator branch 6 are connected by a mutual inductance, and that each arm may be inductive.

Let $\mathbf{E}$ be the electromotive force developed by the generator,

i_g be the current in the detector branch,

$\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, $\mathbf{Z}_5$ and $\mathbf{Z}_6$ be the impedances of the various branches ; and

$\mathbf{W}_{5-6}$ be the conjunctance between branches 5 and 6.

With the detector branch open between b and c the total current is

$$\frac{\mathbf{E}}{\mathbf{Z}_6+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)/(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)}$$

and the drop in potential from d to c is this current times $\mathbf{W}_{5-6}$, or

$$\frac{\mathbf{E}(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)\mathbf{W}_{5-6}}{\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)}$$

The current through branches 3 and 4 is

$$\frac{\mathbf{E}(\mathbf{Z}_1+\mathbf{Z}_2)}{\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)}$$

and the potential drop from a to d is this current times $\mathbf{Z}_3$. The current through branches 1 and 2 is

$$\frac{\mathbf{E}(\mathbf{Z}_3+\mathbf{Z}_4)}{\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)}$$

and the potential drop from a to b is this current times $\mathbf{Z}_1$.

The potential difference across the break between b and c is the potential drop from a to d plus the potential drop from d to c minus the potential drop from a to b, and, with the detector branch closed, the current is that which would be produced by an equal electromotive force in the detector branch, or

$$i_g=\frac{\mathbf{E}}{\mathbf{Z}_m\mathbf{Z}_n^2}[(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)\mathbf{W}_{5-6}+\mathbf{Z}_2\mathbf{Z}_3/\mathbf{Z}_1\mathbf{Z}_4] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $\mathbf{Z}_m$ is the impedance of the detector circuit, and

$$\mathbf{Z}_n^2=\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)$$

It should be noted that to this point no approximations have been made or conditions imposed. However, $\mathbf{Z}_m$ can be expressed readily in terms of $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, etc., only when the bridge is very nearly balanced, or when the impedance of the generator circuit is very high in comparison with the impedance of the arms of the bridge. Then $\mathbf{Z}_m$ may be considered to be the same as though the generator branch were open, in which case

$$\mathbf{Z}_m=\mathbf{Z}_5+(\mathbf{Z}_1+\mathbf{Z}_3)(\mathbf{Z}_2+\mathbf{Z}_4)/(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4).$$

Should a case arise in which the approximation made would not be justifiable it probably would be possible to measure $\mathbf{Z}_m$ directly.

Problem 8.—To determine the current through the detector branch of the Anderson bridge [24] when each branch is inductive. Referring to Fig. 8, let

$\mathbf{E}$ be the potential drop from a to b with the detector branch open,

$\mathbf{i}_g$ be the current through the detector branch, and

$\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, $\mathbf{Z}_5$, $\mathbf{Z}_6$ and $\mathbf{Z}_7$ be the impedances of the various branches.

With the detector branch open, it will be seen by inspection that the potential drop from a to c is

$$\mathbf{e}_1=\frac{\mathbf{E}\,\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}$$

The current through $\mathbf{Z}_4$ is

$$\frac{\mathbf{E}}{\mathbf{Z}_4+\mathbf{Z}_3(\mathbf{Z}_6+\mathbf{Z}_7)/(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}=\frac{\mathbf{E}(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}{\mathbf{Z}_4(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)+\mathbf{Z}_3(\mathbf{Z}_6+\mathbf{Z}_7)}$$

and the part of this current passing through $\mathbf{Z}_7$ is

$$\mathbf{Z}_3/(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)$$

Therefore the current through $\mathbf{Z}_7$ times $\mathbf{Z}_7$, which is the drop in potential from a to d, is

$$\mathbf{e}_2=\frac{\mathbf{E}\,\mathbf{Z}_7\mathbf{Z}_3}{\mathbf{Z}_3(\mathbf{Z}_6+\mathbf{Z}_7)+\mathbf{Z}_4(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}$$

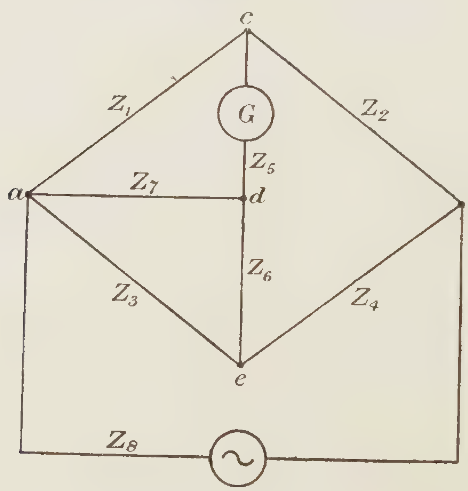

FIG. 8.—ANDERSON BRIDGE.

With the detector branch closed the current in it is e_1-e_2, considered as an electromotive force, divided by the impedance $\mathbf{Z}_m$ of the detector circuit, or

$$\mathbf{i}_g=\frac{\mathbf{E}}{\mathbf{Z}_m}\left[\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}-\frac{\mathbf{Z}_7\mathbf{Z}_3}{\mathbf{Z}_3(\mathbf{Z}_7+\mathbf{Z}_6)+\mathbf{Z}_4(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}\right]$$

If the bridge is very nearly balanced, which is the only case of practical importance,

or if the impedance of the generator branch is very small in comparison with the impedance of the arms of the bridge, in writing an expression for $\mathbf{Z}_m$ the branch points a and b may be considered to be connected by a conductor of negligible impedance. Referring to Fig. 9, it will be seen by inspection that

$$\mathbf{Z}_m=\mathbf{Z}_5+\frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}+\frac{\mathbf{Z}_7[\mathbf{Z}_3\mathbf{Z}_4/(\mathbf{Z}_3+\mathbf{Z}_4)+\mathbf{Z}_6]}{\mathbf{Z}_7+\mathbf{Z}_3\mathbf{Z}_4/\mathbf{Z}_3(+\mathbf{Z}_4)+\mathbf{Z}_6}$$

The suggestion made with reference to the determination of the impedance of the detector circuit in connection with Problem 7 also applies here. Further, it would be possible in any case by transforming both the delta $c\,a\,b$ and the delta $a\,e\,d$ to equivalent three-point stars to write from inspection an expression for the impedance of the detector circuit. In case $\mathbf{E}$ represents the electromotive force developed by the generator instead of the potential drop between terminals a and b of the bridge, it may be necessary to take into consideration the impedance of the generator branch. This adds some complications to the equations without serving any very useful purpose.

To show the applicability of the principle under consideration to a still more

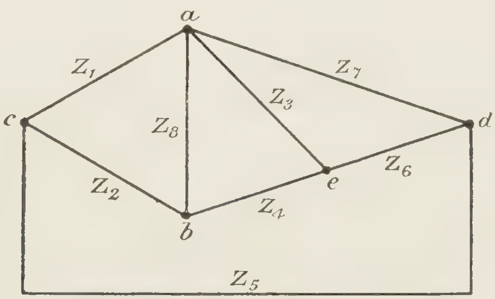

FIG. 9.—ANDERSON BRIDGE ARRANGED FOR DETERMINING IMPEDANCE OF DETECTOR CIRCUIT.

complicated problem, let it be assumed that the currents are transient or discontinuous functions of time.

Problem 9.—To determine the average current in the galvanometer branch of a Maxwell commutator bridge[(25)] for which the circuit arrangement is that shown in Fig. 10. Let

C be the capacitance of the condenser,
A, B and Y be the resistances of the bridge arms,
G be the resistance of the galvanometer branch, and
H be the resistance of the battery branch.

Also let

E be the electromotive force of the battery, and
n be the number of times the condenser is charged and discharged (by the action of the commutator) per second, or the number of cycles per second.

With the galvanometer branch open, if conditions are such that steady states, following changes in position of the commutator, are reached in times less than half

the period, it may be seen that at the point in each cycle at which the condenser branch is opened the condenser is charged to a potential difference of

$$E(A+B)/(A+B+H)$$

Therefore the average current through the condenser and Y branches is

$$nCE(A+B)/(A+B+H)$$

This current causes an average potential drop in the Y branch

$$e_y = YnCE(A+B)/(A+B+H) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and in the battery branch of

$$HnCE(A+B)/(A+B+H)$$

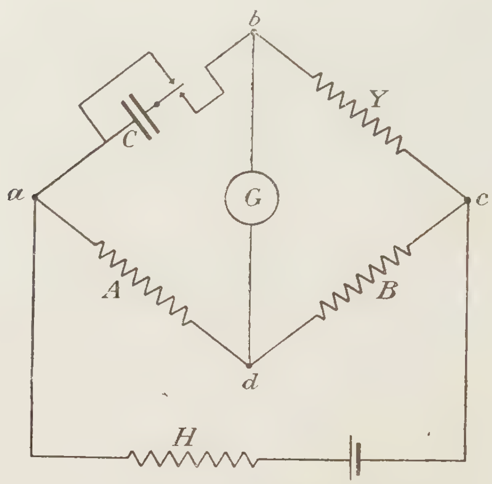

FIG. 10.—MAXWELL COMMUTATOR BRIDGE.

Consequently the average current in the A and B branches is

$$[E-HnCE(A+B)/(A+B+H)]/(A+B+H)$$

and the average potential drop in the B branch

$$e_b = B[E-HnCE(A+B)/(A+B+H)]/(A+B+H) \quad . \quad . \quad . \quad . \quad (2)$$

The average drop in potential across the break in the galvanometer branch, considering the direction dGb as positive, is

$$e_g = e_b - e_y$$

or

$$e_g = \frac{E}{A+B+H}\left\{B - nCY(A+B)\left[1 + BH/Y(A+B+H)\right]\right\} \quad . \quad . \quad . \quad . \quad (3)$$

while during the greater part of each cycle, including the instant when the condenser branch is opened by the commutator, the potential drop across the break is

$$e_g' = EB/(A+B+H) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To determine the average current in the galvanometer branch resulting from

an electromotive force in this branch, equal to the potential drop which would appear across a break were the branch opened, it will be convenient to replace the delta ***d c a*** of Fig. 11, which is equivalent to Fig. 10, by a star, as shown in Fig. 12. In this transformation[11]

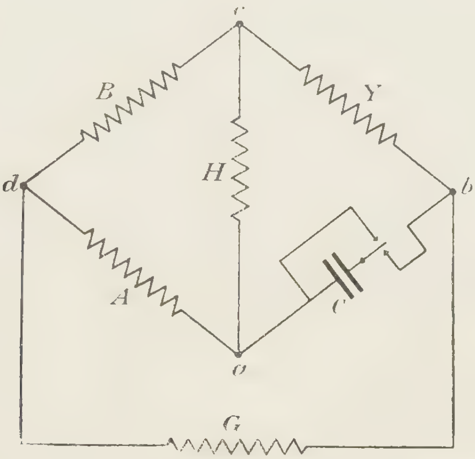

FIG. 11.—MAXWELL COMMUTATOR BRIDGE ARRANGED FOR DETERMINING THE CURRENT RESULTING FROM AN ELECTROMOTIVE FORCE IN THE GALVANOMETER BRANCH.

$$\left.\begin{aligned} P&=BA/(A+B+H)\\ Q&=HB/(A+B+H)\\ S&=HA/(A+B+H) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (5)$$

and

Referring to Fig. 12, it will be seen that at the instant in each cycle at which the condenser branch is opened an electromotive force e_g' in the galvanometer branch would cause a current

$$e_g'/(G+P+Q+Y)$$

in Q and Y, and therefore cause the condenser to be charged to a potential difference of

$$e_g'(Q+Y)/(G+P+Q+Y)$$

The average current through the condenser and S branches which would result from an electromotive force e_g' in the galvanometer is

$$i_c=nCe_g'(Q+Y)/(G+P+Q+Y) \quad \ldots\ldots\ldots \quad (6)$$

and this causes an average potential drop in G and P of

$$(G+P)nCe_g'(Q+Y)/(G+P+Q+Y)$$

Therefore the average current in Q and Y which would result from an average electromotive force e_g in the galvanometer branch which at the instant the condenser branch is opened is equal to e_g' is

$$i_y=\frac{e_g-(G+P)nCe_g'(Q+Y)/(G+P+Q+Y)}{G+P+Q+Y} \quad \ldots\ldots\ldots \quad (7)$$

The total average current in the galvanometer branch would be $i_g=i_c+i_y$, so that

$$i_g=\frac{nCe_g'(Q+Y)+e_g-(G+P)nCe_g'(Q+Y)/(G+P+Q+Y)}{G+P+Q+Y}$$

or

$$i_g=\frac{e_g+nCe_g'(Q+Y)^2/(G+P+Q+Y)}{G+P+Q+Y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Replacing P and Q by their equivalents as given by equation (5) and e_g and e_g' by their equivalents as given by equations (3) and (4) gives

$$i_g=E\frac{\left\{B-nCY(A+B)\left[1+\frac{HB}{Y(A+B+H)}-\frac{B^3H^2/(A+B+H)+2B^2HY+BY^2(A+B+H)}{Y(A+B)[(G+Y)(A+B+H)+B(A+H)]}\right]\right\}}{(G+Y)(A+B+H)+B(A+H)} \quad (9)$$

This gives as the condition of balance—that is, for $i_g=0$,

$$C=\frac{B}{nY(A+B)\left\{1+\frac{BH}{Y(A+B+H)}-\frac{B^3H^2/(A+B+H)+2B^2HY+BY^2(A+B+H)}{Y(A+B)[(G+Y)(A+B+H)+B(A+H)]}\right\}} \quad (10)$$

which differs in form only from the equation given by Thomson.[26] However, the supposition that a steady state is reached in a time less than half a cycle deserves

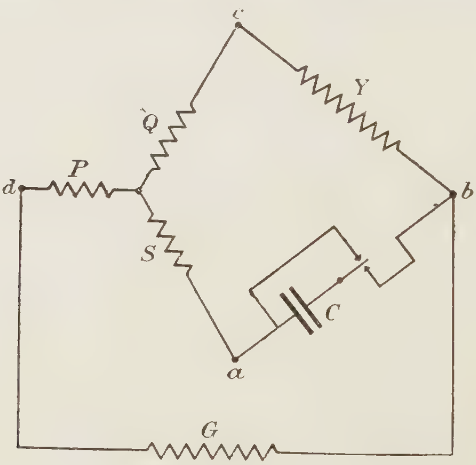

FIG. 12.—MAXWELL COMMUTATOR BRIDGE WITH THE DELTA dca OF FIG. 11 REPLACED BY THE EQUIVALENT STAR.

consideration. The deflection of the galvanometer neither follows the current nor remains stationary. Consequently, there is an electromotive force generated in the galvanometer branch. The effect of this generated electromotive force is not included in equation (10), though the method of analysis followed lends itself readily to its determination. Further, there are reasons why another supposition generally made—namely, that a zero average deflection of the galvanometer is an indication that the average current through it is zero—should not be accepted without experimental proof.

The problems considered should furnish a fairly definite idea as to the possible applications of the principle, while a comparison of the solutions given here with the solutions for the same problems as given in text-books, or as derived by any one of the more usual procedures, should show the advantage to be gained by its use.

Before bringing this Paper to a close, I wish to state a personal point of view regarding it. As I consider the matter, there is only one law, principle or theorem giving the relation between the electromotive forces and the currents in a system of conductors, if of the usual type, which should be considered as fundamental. This was first definitely stated by Ohm approximately 100 years ago, and is generally known as Ohm's law. This Paper was written not so much for the purpose of setting forth a new or little understood principle as of describing a procedure which I have been using in the application of Ohm's law to various problems, and of showing that this procedure possesses certain advantages in comparison with other procedures followed in the application of the same law to the same problems. The form in which the subject is presented here was adopted in the hope of bringing this procedure to the attention of undergraduate students of physics, engineers and those of us who either never have had or have forgotten that expert knowledge of determinants necessary for the expeditious handling of such problems by the "classical" method.

IV. Summary.

1. A brief résumé is given of the procedures which have been proposed for determining the distribution of current in systems of linear conductors. In this connection reference is made to practically all of the laws, principles and theorems which pertain to this field of investigation.

2. A new, or at least not generally known, principle is discussed. This principle may be stated as follows: In a system of linear conductors in which the current in every branch is proportional to the impressed electromotive force, the current in any branch is that which would result should an electromotive force, equal to the potential difference which would appear across the break were the branch opened, be introduced into the branch and all other electromotive forces be removed.

3. A proof of this principle is given, and it is shown that—in some cases, at least—its use leads more directly to a solution for the current in a branch of a network than does the procedure generally referred to as the use of Kirchhoff's laws.

V. Appendix.

Specification for the System of Conductors.

The following is a more complete specification for the system of conductors than that given on page 126.

1. Each branch may contain a resistance, an inductance or a capacitance, or any two or more of these in series; a capacitance between parts of itself; may be inductively related to one or more of the other branches of the system; may be of sufficient cross-section so that the distribution of the current depends upon the time function of the impressed electromotive force; and may be so arranged as to move with respect to a constant magnetic field, thus developing a counter electromotive force. An example of the latter is a branch containing a galvanometer, a telephone receiver or similar apparatus.

2. The impressed electromotive forces may be arbitrary functions of time, and may have an arbitrary distribution throughout the system. For example, in one branch there may be a direct and constant electromotive force, in a second there may be an alternating electromotive force, in a third there may be an alternating electromotive force having an arbitrary phase relation to that in the second or a different frequency, in a fourth there may be a transient electromotive force, etc., or any two or more of the electromotive forces may be the same function of time or may be located in the same branch.

3. The current in each branch and the potential difference between each pair of points depends linearly upon all the applied electromotive forces. By this it is to be understood that should the introduction of an electromotive force $EF(t)$ into a particular branch cause changes in the currents amounting to $I_1F_1(t)$, $I_2F_2(t)$, $I_3F_3(t)$, etc., in the various branches, and cause changes in the potential differences amounting to $V_1f_1(t)$, $V_2f_2(t)$, $V_3f_3(t)$, etc., between various pairs of points, the introduction of an electromotive force $nEF(t)$ into the particular branch would cause changes in the currents amounting to $nI_1F_1(t)$, $nI_2F_2(t)$, $nI_3F_3(t)$, etc., in the same branches, and cause changes in the potential differences amounting to $nV_1f_1(t)$, $nV_2f_2(t)$, $nV_3f_3(t)$, etc., between the same pairs of points. However, one branch, if it is the only one in which the current is to be determined, may be of such a type as to cause a departure from proportionality between currents, potential differences and the applied electromotive forces.

For such a system of conductors, the principle and the proof for it applies as readily as they do when each branch contains resistance only, and all currents and electromotive forces are direct and constant.

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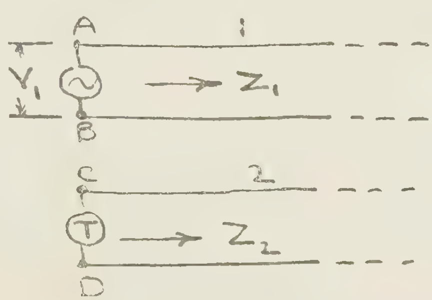
DISCUSSION.

Mr. ROLLO APPLEYARD: Dr. Wenner's paper is a further elucidation of a theorem due to L. Thévenin (*Comptes Rendus* Vol. 97, 1883), and explained and generalised in relation to modern problems by K. S. Johnson (*Transmission Circuits for Telephonic Communication*, p. 79). The examples given by Dr. Wenner are valuable testimony of the simplicity of the method. To obtain confidence it is useful to begin with the case of a circuit consisting of a battery connected to a shunted galvanometer. First express the current in the battery-branch of resistance r, as the product of the galvanometer-current and the multiplying-power $(g+s)/s$, of the shunt. Then express the same current directly from Ohm's law—i.e., $E\Big/\left(r+\frac{gs}{g+s}\right)$. Equate these currents, and derive the galvanometer-current as the ratio of $E[s/(g+s)]$ to $[r+gs/(g+s)]$, which is the same as the ratio of $E[s/(r+s)]$ to $[g+rs/(r+s)]$. This is seen to be the ratio of the potential-difference that would exist across the galvanometer-terminals if the galvanometer-branch were cut, to the resistance made up of the galvanometer and the shunted battery circuit when the battery is replaced by a resistance equal to the battery-resistance, the electromotive-force of the battery being imagined suppressed. By adding branches one at a time, a complex network can be treated in like manner, and assurance is gained that to find the current in a given branch it suffices (1) to suppose the branch cut; (2) to express the potential-difference between the terminals of the cut branch; (3) to express the total resistance made up of the branch itself, and of the resistance between its terminals when the branch is cut and the electromotive-forces are suppressed; (4) to divide the result of (2) by the result of (3). In the generalisation, capacities, inductances and leakances can be treated as resistances by the well-known device.

Mr. G. W. SUTTON questioned the value of the method for teaching purposes. The old methods were admittedly more laborious, but it was in practice necessary to limit the number of principles which have to be explained to an ordinary physics class.

Dr. A. RUSSELL said that in his opinion the author's method gives certain classes of results in a tenth of the time taken by direct methods.

Dr. A. ROSEN (communicated): The theorem given by Dr. Wenner can be stated in a slightly different form as follows: The current in any branch of a network may be determined by replacing the remainder of the network by a generator whose E.M.F. is V_g and internal



impedance Z_g; V_g and Z_g are the values measured at the terminals when the branch is opened, i.e., the generator is on open circuit. The current through the branch of impedance Z is $\frac{V}{Z+Z_g}$.

In this form the case is analogous to that of a real generator, which helps to make it easier to remember and apply. The theorem is useful in the theory of telephone transmission, and I am indebted to Mr. R. Haemers, of the Belgian Post Office for the following illustration :—

The interference between the circuits 1 and 2 in the figure is caused by a complex series of electrostatic and electromagnetic couplings, and is measured by the ratio of the power in the disturbed to that in the disturbing circuit when the former is closed by an impedance equal to its own, i.e., Z_2. The interference may be expressed as an attenuation factor b, where $\frac{P_2}{P_1} = e^{-2b}$. It is required to find the current in the telephone T when there is a voltage V_1 across AB. Let the p.d. at the terminals of circuit 2 when the branch is open be V_0. Then when closed by Z_2 the current is $\frac{V_0}{2Z_2}$. Hence $P_1 = \frac{V_1^2}{Z_1}$, $P_2 = \frac{V_0^2}{4Z_2}$ and $e^{-2b} = \frac{V_0^2}{4V_1^2} \cdot \frac{Z_1}{Z_2}$. When closed by a telephone of impedance Z_t, the current is $I_t = \frac{V_0}{Z_2+Z_t} = \frac{2V_1}{Z_2+Z_t} \cdot \sqrt{\frac{Z_2}{Z_1}} \cdot e^{-b}$

Mr. A. Campbell (communicated) : All who are interested in electrical measurements are deeply indebted to Dr. Wenner for the many illuminating and helpful papers which he has already published ; the present Paper adds to the list and it is clear, from the many interesting examples which he deals with, that his system of working will be most useful to many workers who do not care for long lines of simultaneous equations. (Could Dr. Wenner kindly apply it to the Carey-Foster network ?) To test the working of the system I looked into Problem 6. It may be worked out in the ordinary way as follows :—

Let g = resistance of instrument,
i = current through instrument,
and I = total current through A.

We are given that

$$(g+20)i = 0{\cdot}15 \qquad (1)$$

and

$$AI + gi = 1{\cdot}5 \qquad (2)$$

We have also

$$\frac{i}{I} = \frac{B}{B+g} \qquad (3)$$

By eliminating i and I we obtain

$$AB - 200B = g(9B - A).$$

If this condition is to be true *for all values of g* then

$$AB - 200B = 0$$

and

$$9B - A = 0$$

or

$$A = 200 \text{ ohms},$$

and

$$B = 22{\cdot}22 \text{ ohms},$$

the result that Dr. Wenner obtains in two lines ! The result in itself is interesting, for it holds good no matter what resistance the instrument may have.

X.—A CAPACITANCE BRIDGE OF WIDE RANGE AND A NEW INDUCTOMETER.

By ALBERT CAMPBELL, *M.A.*

Received November 16, 1926.

ABSTRACT.

A bridge is described by which quick measurements can be made of capacitances covering a range of from $1\mu\mu$F up to 30μF, the power factor also being indicated. The unknown capacitance C is put in parallel with a resistance P, and the effective self-inductance of the combination, which is approximately equal to $-P^2C$, is read on a mutual inductometer forming part of the bridge. By giving P a series of suitable values scale multipliers providing for a very wide range of capacitance are obtained. The inductometer used is of a new type, having a circular scale extending to about 260°, the percentage accuracy of reading being almost constant over the greater part of the range. This novel scale system allows the lower readings to be taken with good accuracy. A small rheostat allows the power loss in the condenser to be balanced, and enables the power factor to be deduced.

§ 1. INTRODUCTORY.

FOR the accurate measurement of capacitance the author many years ago* introduced Heydweiller's modification of Carey Foster's method, using as standard a mutual inductometer calibrated against a mutual inductance calculated from its dimensions. This method still holds its place as one of the best for precision measurements of capacitance and power factor ; but it requires a rather large set of resistances if a wide range of values are to be dealt with. For tests which do not require the highest accuracy the author has designed a simple self-contained bridge which affords a quick and convenient means of measuring capacitances over a long range ($1\mu\mu$F up to 30μF), also indicating the power factor with fair accuracy except in extreme cases.

§ 2. THEORY OF THE METHOD.

The method consists in placing the unknown condenser C in parallel with a known non-inductive resistance P, and reading the effective self-inductance and resistance of the combination by means of a bridge containing a mutual inductometer. Let the combination be represented as in Fig. 1, where the series resistance x causes the power loss in the condenser.

Thus, if the pulsatance $\omega=2\pi f$, where f is the frequency, the effective resistance P' and inductance L' are given by the equations

$$P'=\frac{P+Px(P+x)C^2\omega^2}{1+(P+x)^2C^2\omega^2} \quad \text{. . . . .} \quad (1)$$

and

$$L'=\frac{-P^2C}{1+(P+x)^2C^2\omega^2} \quad \text{. . . . .} \quad (2)$$

In most cases in practice all the terms in ω^2 are much smaller than 1, and then the approximate values are

$$P' \doteq P[1-P(P+x)C^2\omega^2] \quad \text{. . . . .} \quad (3)$$

and

$$L' \doteq -P^2C[1-(P+x)^2C^2\omega^2] \quad \text{. . . . .} \quad (4)$$

* Proc. Phys. Soc., Vol. 20, p. 626 (1907).

When ω, P, P' and L' are known, these equations suffice to determine C and x, and hence the power factor $Cx\omega$. By suitably choosing ω and P the term in ω^2 in equation (4) can usually be made negligible (except for very leaky condensers), and then we have

$$L' = -P^2C, \qquad (5)$$

or,

$$C = L/P^2, \qquad (6)$$

where

$$L = -L'.$$

Thus the capacitance C is obtained directly by multiplying the reading L of the inductometer by the factor $1/P^2$. This factor is conveniently chosen to be some power of 10, so making the scale read the capacitance practically directly.

Using the approximation of equation (6) we have from equation (4)

$$\text{Power factor} = Cx\omega,$$

$$\doteq \frac{P - P'}{L\omega} - CP\omega \qquad (7)$$

Unfortunately, as in some of the other methods (e.g., Carey Foster's), the power factor, if very small, comes out as the difference of too much larger quantities.

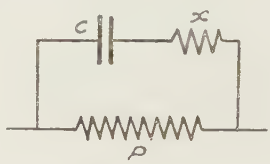


Fig. 1.

The present method, however, has been designed for the convenient determination of capacitance, and treats power factor only as a secondary consideration.

If the resistance P is not non-inductive, but has a small self-inductance l, then, when $s = o$, we have

$$P' = \frac{P}{(1 - \omega^2 lC)^2 + P^2C^2\omega^2} \qquad (8)$$

and

$$L' = \frac{l(1 - \omega^2 lC) - P^2C}{(1 - \omega^2 lC)^2 + P^2C^2\omega^2} \qquad (9)$$

In the actual bridge, however, l can be kept so small that these equations are unnecessary in general.

§ 3. General Arrangement of the Bridge.

The bridge (in the present model) is arranged, as in Fig. 2, with equal ratio arms ($R = S$), P and Q being equal resistances in the other arms, which also include H and K, the two sections of the secondary winding of the mutual inductometer, which are made identical in resistance and self-inductance. The primary circuit of the inductometer enters the upper corner of the bridge through the slider of the potentiometer rheostat B, which has sufficient range to balance the small change in effective resistance when the condenser C is put across the terminals of P (or of Q). The inductometer scale is marked to read L directly, usually extending from -10 to $+105$ microhenries. In the actual instrument the resistances P and Q can be

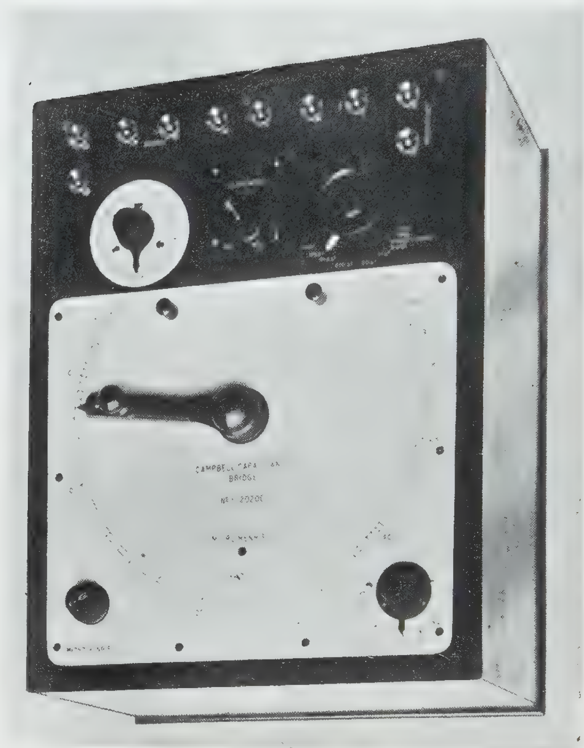


FIG. 3.—CAMPBELL CAPACITANCE BRIDGE

[*To face page* 147

simultaneously altered by a step-switch, giving a series of values, 3·162, 10, 31·62, 100, 316·2, and 1,000 ohms, which give scale ranges of $100\mu\mu$F, $1{,}000\mu\mu$F . . . up to 10 microfarads.

Two fixed steps on the inductometer carry each range to three times the above, giving a total range from (say) $1\mu\mu$F to 30μF. The rheostat B has its scale marked to read directly changes in P or Q, and can be shunted so as to increase the accuracy of reading tenfold. To make a measurement, the range-setting switch is turned to give the desired range, and a balance (in the telephone or vibration galvanometer G) is obtained by setting B and a small zero-setting L-dial, the main L-dial reading zero. The condenser C is then put across P, and a new balance obtained by altering B and the main inductometer. Then the multiplied inductometer reading gives C, and the change in the rheostat reading gives $(P-P')$, and hence the power

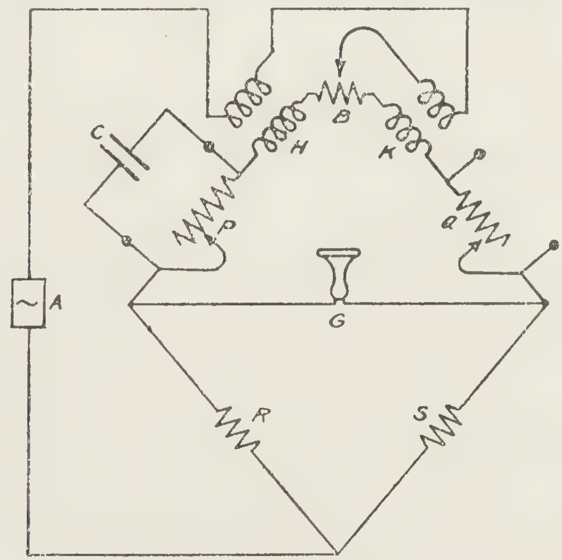


Fig. 2.—Capacitance Bridge.

factor. In many cases the junction of the ratio arms is a convenient point to earth.

Two condensers not very unequal can be compared by putting the larger across P and the smaller across Q.

The actual instrument, which is made by Messrs. The Cambridge Instrument Co., is illustrated in Fig. 3.

§ 4. A New Inductometer.

The accuracy of measurement in this capacitance bridge has been greatly increased by the use of a new type of mutual inductometer designed by the author.*
The scale of this new type embodies two improvements :—

(1) Its angular extent is very great, being about 260°, older types giving only about 160°.

* A. Campbell, British Patent Specifications, 244,596 and 252,990.

(2) The scale gradually opens out towards the zero, in such a way that over the greater part of the range *the percentage accuracy of reading is nearly constant.* From Fig. 4 it will be seen how nearly this condition is fulfilled from 10 up to 100 μH.

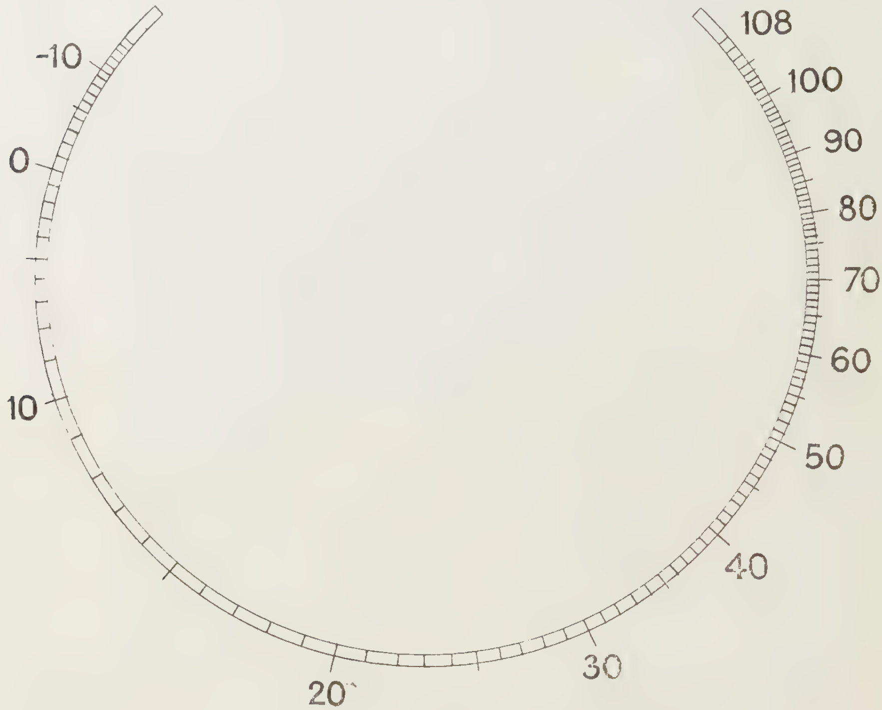


FIG. 4.—SCALE OF NEW CAMPBELL M INDUCTOMETER.

there is a slight intentional opening towards the top, in order not to have the divisions too close there, the minimum length being 1·8 mm. per division.

The importance of constant percentage accuracy of reading does not seem to

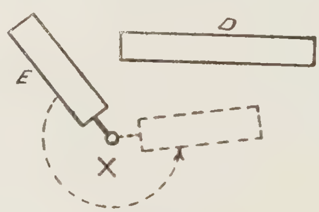


FIG. 5.

have been realised by many instrument makers and users, and it is very often difficult of attainment. The equation for a scale of constant percentage accuracy is

$$M = a\varepsilon^{\theta},$$

where M is the mutual inductance at angular reading θ, ε the base of natural logarithms, and a a constant; but such a scale could never get down to zero until

$\theta = -\infty$. So, for a practical scale, we must be content to close it in gradually at the lower end so as to reach zero and negative values.

In the present instance, the ideal scale has been attained by constructing the inductometer as shown in plan in Fig. 5, where the fixed and moving circuits are the vertical coils D and E respectively, E being mounted on a vertical axle X carrying the pointer.

§ 5. Limits of Accuracy.

In the Capacitance Bridge, except at the beginning of the very lowest range, the accuracy of reading is within 2 parts in 1,000 throughout. With the two highest ranges (or for very large power factors) the correction shown in equation (3) is sometimes required. For example, with frequency of 800 ∾ per second for $C=1\mu$F, the correction is about 2 in 1,000, and for $C=30\mu$F it becomes quite excessive. This difficulty with the very large condensers can be avoided by making the tests at a lower frequency, such as 100 ∾ per second. In conclusion, it may be remarked that the system of shunting used in the bridge involves considerable loss of sensitivity, but this disadvantage appears to be well outweighed by the convenience and ease of working of the instrument.

DISCUSSION.

Mr. D. W. Dye said that it seemed paradoxical at first sight that a high impedance could be more accurately measured when diluted with a shunt, but, as in a method described by himself in which the impedance to be measured is shunted by a resistance of 100,000 ohms, the dilution had the effect of making an accurate measurement practicable. The new scale of the instrument, which was substantially that of a slide rule, was particularly valuable, and might be applied with advantage to other instruments. Would it be possible to make the inductometer astatic, in order to avoid the errors introduced by variation of the earth capacity of the telephones ?

Dr. E. H. Rayner said that the proposed method of measuring power factor was particularly interesting in view of the importance of the power factor in determining the behaviour of cables under high voltages. The volt-amperes applied to an ordinary cable circuit amount to about 200 times the real power, the power factor being nearly 90 degrees, and in measuring this factor a sensitivity of 0·0001 radian at 20,000 volts is desirable.

Mr. Rollo Appleyard said that the instrument would be of great value to electricians if it could be had at a moderate price. The scale was that of the aneroid barometer, and in effect it was the same scale as that originally used by Cavendish when he first compared the capacity of a battery of Leyden jars with that of a small plate condenser by repeated sharing of charges.

Mr. G. W. Sutton asked whether it would be possible to design the instrument so as to avoid the effect of capacity between the primary and secondary of the mutual inductance.

Author's Reply.—In reply to Mr. Dye: The procedure in my method of testing condensers is quite the same as that employed in Mr. Dye's method of testing large self-inductances (which, I may remark, is probably the most accurate of all known methods for that purpose) ; but the two methods depend on quite different formulæ. Thus, the condenser method is practically direct reading and independent of frequency, while in the self-inductance method the results have to be deduced by calculation involving the frequency.

With regard to astaticism, it is not very difficult to obtain moderate astaticism in an inductometer of the new type, but I think that astaticism almost always involves loss of efficiency (e.g., by increasing the weight of wire used). In the present model of the capacitance bridge, I have not found the effect of stray fields troublesome, so long as the generator is kept sufficiently distant. In reply to Mr. Sutton, I would point out that in the capacitance bridge the maximum mutual inductance used is of the order of 50μH, and hence the internal capacitances of the instrument have very little effect. In higher-reading inductometers of the new type the troublesome effects of capacitance are minimised by suitable design of the coils. It is interesting to learn from Mr. Appleyard's remarks that the ordinary aneroid barometer has a scale of constant percentage accuracy ; this appears to be due to its inherent law of working, while my inductometer scale is the result of design involving a great number of experiments. I would thank all the speakers for their kind remarks of appreciation.

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XI.—ON THE SPECTRA OF DOUBLY-IONISED GALLIUM AND INDIUM.

By K. R. Rao, *M.A.*, Madras University Research Scholar.

Received October 20, 1926.

(*Communicated by* Prof. A. Fowler, *F.R.S.*)

Abstract.

The Paper gives an account of experimental work which has led to an extension of our knowledge of the second spark spectra of In and Ga, into the region of long wave-lengths. After making a careful study of the spark spectra of the elements in hydrogen and air under different conditions of excitation the author has identified the lines corresponding to the second series in the spectra of doubly-ionised Indium and Gallium. The principal features of the spark spectra are now known, and all these structures are in beautiful accord with the quantum theory of spectral line emission.

Introduction.

According to Bohr's theory of spectra and atomic structure the arc and spark spectra of an element are quite distinct, arc lines being emitted by the interorbital transitions of the valence election in the neutral atom, while spark lines result in the energy changes in atoms that have lost one or more of their outermost electrons. Recent researches of Fowler and Paschen have shown that, in passing from arc to spark lines, the series constant is changed from N to $4N$, $9N$, etc., quite in keeping with Bohr's theory.

One of the main approaches to the problem of atomic structure is by the identification of wave-lengths in arc and spark spectra of different elements, and their representation by formulæ. The main difficulty in this work is that in general the experimental arc and spark spectra of any particular element contain many lines in common ; the true spark lines are often excited in an ordinary electric arc, while arc lines appear also in high potential sources such as condensed sparks.

According to the spectroscopic displacement law the doubly-ionised spectra of Ga and In are expected to consist of series of doublets with a Rydberg constant $9N$. Owing to the high value of the constant the primary series fall in the extreme ultra-violet, while the secondary series lines fall in the region above $\lambda 2200$. After the publication of the work of Weinberg(1) on the spark spectra of Ga and In, and of Lang(2) on Tl, a study of the series in the spectra of doubly-ionised Ga,(3) In and Tl was undertaken. When a partial analysis of the second spark spectra of Ga and In had been made, Carroll communicated a Paper(6) on "Vacuum Spark Spectra of some of the heavier elements and series classifications in the spectra of ionised atoms homologous with Cu, Ag and Au." As the classifications of Carroll were mainly confined to the primary series, the author's efforts were concentrated on the study of the spark spectra in the visible and ultra-violet regions up to $\lambda 2200$, to identify the wave-lengths of Ga III and In III, and to establish, if possible, other series relationships. Though only a few leading members of the series have so far been detected, it is thought desirable to publish the present account, as it might possibly facilitate the investigation of series in the spark spectra of succeeding elements of the periodic table.

Experimental Procedure.

To provide data likely to be useful in identifying the spectra of these elements at higher stages of ionisation, a study was made of the spark spectra of the elements in hydrogen, in air, and in vacuum, under different conditions of excitation. In the case of Indium the arc in vacuum also was examined. The metal in each case was contained in a small cavity made at the end of an aluminium rod, which served as one electrode, while the other electrode was a second aluminium rod made to taper at the end. The spectra were photographed before and after the introduction of the metal, and these helped in the elimination of lines due to aluminium or other impurities. To obtain the enhanced spectra of these elements a large X-ray coil capable of giving a 10 in. spark in air was used, employing a mercury interrupter in the circuit, and the primary was fed with a current of 15 amps. The secondary contained a battery of large Leyden jars of capacity about 0·02 mfd. in parallel with the spark-gap. An auxiliary spark-gap in air was also placed in series with this. In order to classify the lines and assign them to the various stages of ionisation of the element in question the degree of excitation was largely varied. This was attained by inserting a varying self-induction in the secondary circuit or changing the condenser capacity, or adjusting the pressure of the hydrogen in the spark chamber. Certain strong lines cease to be produced with the smaller intensities of discharge, and, as such, are assigned to higher stages of ionisation. The effect of self-induction is clearly seen from the Plates 1 and 2. In general the lines are intense and diffuse in the spark in air spectra, while in an atmosphere of hydrogen they are sharp and well-defined.

To investigate the region from $\lambda 6000$ to $\lambda 3500$ an ordinary Hilger constant deviation spectrograph and a plane grating spectrograph of about 15000 lines to the inch were used, while for the ultra-violet a large Hilger quartz spectrograph was utilised.

For measuring the wavelengths, the iron arc and Zn-Cd spark were used as standards. In the case of Indium, the wave-length measurements obtained by the author were found to differ from the values given by Schulemann[(4)] consistently by about 0·1 A.U. As Schulemann worked with a 10 ft. concave grating his wave-lengths were taken for the lines referred to. After the completion of this work Carroll's Paper containing wave-lengths for Gallium and Indium appeared. In the case of Ga these recent measurements of Carroll have been substituted in place of the writer's measurements.

Notation and Numeration of Terms.

The notation adopted in the present Paper is generally similar to that used by Prof. Fowler[(5)] in his Paper on ionised oxygen, the respective inner quantum numbers being used as subscripts in the case of multiple terms. The numeration of series terms is given on the new system introduced by Bohr on theoretical grounds. In the consideration of series, however, the azimuthal component of the total quantum number is sufficiently represented by the symbols s, p, etc. These symbols are adopted here for convenience of printing, as all the terms are doublets, but it is to be understood that in the generalised notation, s is equivalent to ${}^2S_1$, p_2 to ${}^2P_2$, and so on.

The Series System of In III.

In accordance with the displacement law the spectrum of doubly-ionised Indium bears a general resemblance to the series of Ag I and Cd II. The lines which are considered to belong to this series are brought together in the table below with their wave-numbers in vac., intensities and separations. The last column gives the designation assigned to each line.

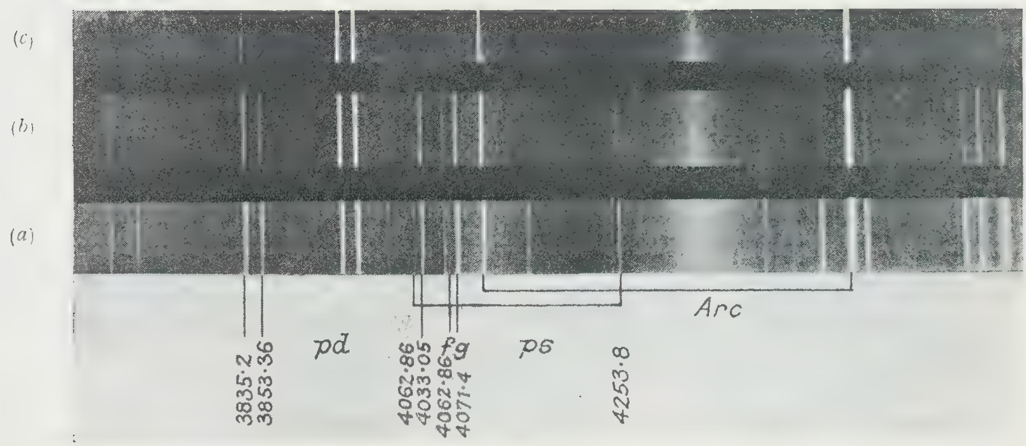
λ I.A.	Int.	ν (vac.)	Δν	Series designation.
5644·86	6	17710·3	1337·4	$6s-6p_1$
5248·52	8	19047·7		$6s-6p_2$
4253·88	5	23501·4	1337·4	$6p_2-7s$
4024·83	4	24838·8		$6p_1-7s$
4062·86	4	24606·3	181·8	$6p_2-6d_2$
4033·05	8	24788·1	1337·8	$6p_2-6d_3$
3853·36	7	25944·1		$6p_1-6d_2$
5918·54	5	16891·4	184·5	$6d_3-5f$
5854·58	3	17075·9		$6d_2-5f$
3008·30	10	33232·0	281·0	$5d_3-4f$
2983·04	8	33513·0		$5d_2-4f$
4071·40	6	24554·0		$4f-5g$

The work of Fowler, Paschen, Millikan and Bowen on series in the spectra of ionised atoms indicates that there is generally a regular displacement of corresponding lines towards the region of shorter wave-lengths in passing from Na I to Cl VII. A comparison with the already located corresponding members of Ag I and Cd II suggested that the pair $6s-6p_{21}$ of the second principal series falls in the region λ5400. Further, an analogy between the doublet separations of Na I, Mg II, etc., and those of Ag I and Cd II led to the following values in the case of In III,

$$5p_1-5p_2=4600 \qquad 6p_1-6p_2=1400$$

Repeated observations of the spark spectra of Indium in the visible region indicated the presence of the pair λ5645 (6) and λ5249 (8) with $\Delta\nu=1337{\cdot}4$. The pair is intense and prominent under the maximum excitation and is easily suppressed by the insertion of self-induction, and, as such, is believed to belong to In III. The relative order of the intensities show that it must clearly be a principal pair. The detection of two other pairs having the above frequency separation and persisting under the same conditions as the above doublet confirmed the identification of the principal pair. The magnitude of the intensities and the presence of the satellite shows that the third doublet in the above table is the first member of the diffuse secondary series. This fixes the separation of the $6d_{32}$ terms of the series. A further search revealed the pair $6d_{32}-5f$ with this separation.

The first fundamental series is generally very prominent in the spectra of ionised atoms, and is perhaps the easiest to fix upon. The f term being "hydrogen-like" is calculable and the sequence of $5d$ terms in Ag I, Cd II, fixes the position of the member $5d_{32}-4f$ in the spectrum of In III. This led to the identification of the only



(a) (b) (c)—Spectra with increasing self-induction.

FIG. 1.

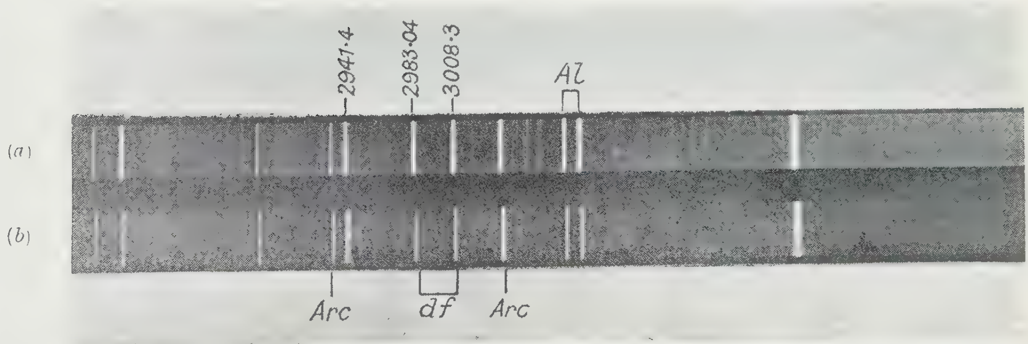


FIG. 2.

PLATE I.—SPARK SPECTRUM OF INDIUM.

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strong pair $\lambda\lambda$3008, 2983, in this region, which, from an examination of the various spectra, seems definitely to belong to In III. No satellite has, however, been detected. The f level may be single or, if double, the levels must be too close to be resolved on these plates. The $4f-5g$ line is next considered. In the appropriate region there is a line λ4071, which is clearly of the second stage of ionisation. If this be $4f-5g$ a probable value of 39600 for $5g$ will lead to a value of the $4f$ term, which, perhaps, may be a little too high in the sequence of terms from Ag I to In III. Still as this line appears along with the other classified pairs, as there is no other line within about 20 A.U., particularly on the long wave-length side of it, which may be said to belong to In III, and as corresponding lines in the case of the elements (Al III and Ga III) of the same sub-group of the periodic table have been excited with sufficient intensity under similar conditions, the above identification has been adopted. This line has not been recorded by Schulemann, but is present on all the Indium plates taken here. The wave-length is carefully measured by taking the oxygen triplet $\lambda\lambda$4075·94, 4072·40, 4069·93 as standards.

Calculation of Series Limits.

Owing, perhaps, to the rapid fall in intensity in the successive members of the series, no observed series is long enough to permit the use of any fairly accurate series formula for the calculation of limits. They are, therefore, determined on the assumption of the values for the remote levels. As Carroll did not fix the super-fundamental series line $4f-5g$, he assumed the value of f for the evaluation of the remaining terms. But as this member is now identified a recalculation of the term values is made by using the value of the more remote level g. The table below gives the term values and the effective quantum numbers.

Designation.	Term Value.	Effective Quantum No.
$5s$	226132	2·089
$6s$	101051	3·126
$7s$	58501	4·108
$5p_2$	164605	2·449
$5p_1$	168950	2·417
$6p_2$	82063	3·469
$6p_1$	83340	3·442
$5d_3$	97387	3·184
$5d_2$	97664	3·180
$6d_3$	57214	4·145
$6d_2$	57396	4·138
$4f$	64154	3·913
$5f$	[40320]	4·955
$5g$	39600	4·999

The value of the limit of $6p$ of the stronger member of the series $6p_{21}-6s$ is calculated from the first two members by the use of a simple Rydberg formula. The adoption of $9N$ for the series constant in the calculation leads to term values

which are consistent with the sequence of terms of Ag I and Cd II. This justifies that the series system belongs to the second stage of ionisation.

	ν
$6p_2-6s$ $6p_2-7s$	$-19047{\cdot}7$ $23501{\cdot}5$

From these, $6p_2=81102$.

This limit agrees fairly with the value of $6p_2$ obtained as above on the assumption of g.

The terms $6s$ and $5p_{21}$ give for the positions of the pair of the first sharp series :—

	ν	Δν
$5p_2-6s$ $5p_1-6s$	63554 67899	4345

A search for this pair is made among the published wave-lengths of vacuum-spark spectra of Indium. No suitable pair is, however, found in the calculated position. The lines $\lambda 1571{\cdot}5$ (1) and $\lambda 1472{\cdot}4$ (5) are in the approximate position, but the intensities and the separation are not satisfactory. The pair,

λ	Int.	ν	Δν
1599·5 1495·1	4 2	62520 66885	4365

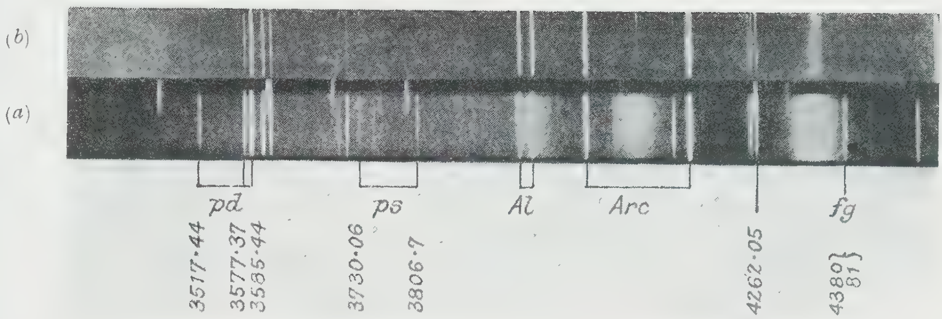
has perhaps the appropriate separation, but it is a little too remote from the calculated position. It may be that the equality of separation is fictitious.

Gallium.

The identification of the series of In III considerably facilitated the working out of the series of doubly ionized Ga. The separation $5p_{21}$ of the principal pair is now approximately estimated to be about 520. The sequence of corresponding members of Cu I, Zn II, and the position of the pairs in In III suggested the classification of the first two pairs in the following table, which is self-explanatory. All the lines are found to belong to Ga III. The diffuse pair and satellite long eluded detection. There are two lines $\lambda\lambda 3577$ and 3517, exactly in the required position, having the separation $\triangle\nu=476$. The intensities, their position and behaviour suggested that they may be classed as $5p_2-5d_3$ and $5p_1-5d_2$. If this be correct, the satellite $5p_2-5d_2$ must have a frequency 27883. This was then searched for.

λ I.A.	Int.	ν (vac.)	Δν	Series Designation.
−4994·15 −4863·19	7 5	−20018·0 −20556·5	538·5	$5p_1-5s$ $5p_2-5s$
3806·72 3730·06	5 4	26261·9 26801·6	539·7	$5p_2-6s$ $5p_1-6s$
(3585·44)* 3577·37 3517·44	... 7 6	(27882·7) 27945·6 28421·7	62·9 539·0	$5p_2-5d_2$ $-5d_3$ $5p_1-5d_2$

* Taken from Carroll's data.



(a)—Without self-induction. (b)—With self-induction.

FIG. 1

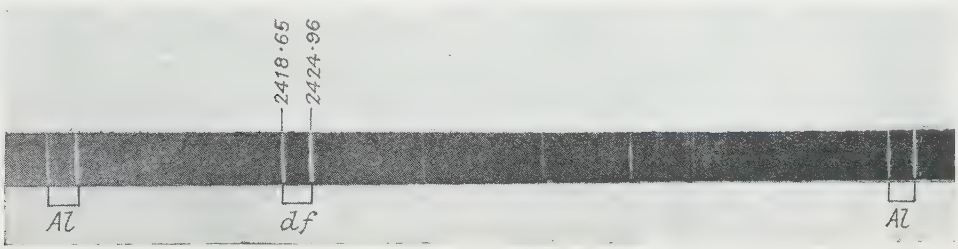


FIG. 2.

PLATE II.—SPARK SPECTRUM OF GALLIUM

To face page 154]

But the line λ3586, being the first fundamental series line of Al II, is found on all plates to be very intense and diffuse and is, therefore, considered to have masked the presence of the faint satellite. The recently published measurements of Carroll[(6)] have confirmed this view. The satellite is included in the table on p. 154.

Series Limits.

As in the case of Indium, no series is extended far enough to justify the use of any accurate formula. The term values are calculated from the frequencies of the members identified by assuming the value of 5s obtained by Carroll from a series of pairs from $4s$–$4p$ to $4f-5g$. The term values and effective quantum numbers are tabulated below.

Designation.	Term Value.	Effective Quantum Number.
$5s$	107064	3·037
$6s$	60245	4·048
$5p_2$	86507	3·378
$5p_1$	87046	3·368
$5d_3$	58561	4·106
$5d_2$	58624	4·104

The value of $5p_2$ thus obtained agrees fairly with the value 87092 got by the application of an approximate Rydberg formula to the first two members of the second sharp series. The adoption of $9N$ for the value of the series constant and the representation of the series system are justified.

Details as to the various series of In III and Ga III which have so far been traced (including the work of Carroll) are given in Tables A and B at the end, arranged on the plan adopted by Fowler in his " Report on Series."

Comparison of Ag I, Cd II, In III and Cu I, Zn II, Ga III.

Doublet Separations.—The table contains the doublet separations for the series of each of these groups of elements of similar electronic structure.

Element.	Atomic Number.	1st p Separation.	2nd p Separation.
Cu I	29	248·0	...
Zn II	30	872·2	245·35
Ga III	31	1716·0	539·0
Ag I	47	920·6	202·9
Cd II	48	2482·8	672·82
In III	49	4345·0	1337·4

The curve connecting $\log \triangle \nu$ and log (squares of net nuclear charges) is found to be very approximately linear.

Millikan and Bowen's application of Sommerfeld's fourth-power law for X-ray doublets to the field of optics shows that the separation is given by $\Delta\nu = \xi \left(\frac{Z-s}{n}\right)^4$ with the usual notation. The variation of the screening constant "s" as we pass from Ag I to In III is illustrated below.

Z	Element.	$\Delta\nu$	$\sqrt[4]{\Delta\nu/0{\cdot}0135}$	s
47	Ag I	202·9	11·08	35·82
48	Cd II	672·82	14·94	33·06
49	In III	1337·4	17·74	31·26

Corresponding Lines.—The regular displacement of corresponding lines in the case of elements of like electronic structure is shown by a comparison of these spectra :—

	$6s-6p_2$	Diff.	$6p_2-6d_3$	Diff.
Ag I	5946	6447	5715	9149
Cd II	12393	6655	14864	9924
In III	19048		24788	
	$5s-5p_2$	Diff.	$5p_2-5d_3$	
Cu I	6246	6928	6008	10374
Zn II	13174	7374	16382	11564
Ga III	20556		27946	

Term Values.—The s and p terms are found to diminish in these sequences as in the case of Na I to Cl VII. But the d terms also are found to regularly decrease.

	$6s$	$7s$	$6p_2$	$5d_3$	$6d$	$4f$
Ag I	18540	9209	12596	12371	6881	6892
Cd II	13347	(7300)	10248	11633	6532	6986
In III	11228	6500	9111	10821	6357	7128
	$5s$	$6s$	$5p_2$	$4d_3$	$5d$	
Cu I	19171	9460	12925	12366	6917	
Zn II	14114	7598	10820	11983	6725	
Ga III	11896	6694	9612	11512	6507	

Comparison of Ga III and In III.

As elements belonging to the same sub-group of the periodic table, they exhibit progressive spectral differences. The doublet separations are roughly proportional to the squares of their atomic numbers. The limits of the sharp and principal series, as is evident from the foregoing tables, are displaced towards the red with increase of atomic number.

TABLE A.—INDIUM.

First Principal Series : $6s=101051$

Classification.	λ I.A.	Int.	ν	Δν	m	mp_{21}
$6s—6p_2$	5248·52	8	19047·7	1337·4	6	82003
$6s—6p_1$	5644·86	6	17710·3			83340

Second Sharp Series : $6p_2=82003$
$6p_1=83340$

						ms
$6p_2—6s$	−5248·52	8	—19047·7	1337·4	6	101051
$6p_1—6s$	−5644·86	6	—17710·3			
$6p_2—7s$	4253·88	5	23501·5	1337·4	7	58501
$6p_1—7s$	4024·83	4	24838·8			

Second Diffuse Series : $6p_2=82003$
$6p_1=83340$

						md_{32}
$6p_2—6d_2$	4062·86	4	24606·3	181·8		57214
$6p_2—6d_3$	4033·05	7	24788·1		6	
$6p_1—6d_2$	3853·36	8	25944·1	1337·8		57396

Second Fundamental : $6d_3=57214$
$6d_2=57396$

						mf
$6d_3—5f$	5918·54	5	16891·4	184·5	7	[40320]
$6d_2—5f$	5854·58	3	17075·9			

First Principal Series : $5s=226132$

	λ (vac.)					mp_{21}
$5s—5p_2$	1625·3*	15	61527	4345	(5)	164605
$5s—5p_1$	1748·8*	15	57182			168950

First Diffuse Series : $5p_2=164605$
$5p_1=168950$

						md_{32}
$5p_2—5d_2$	1494·0*	3	66934	284		97387
$—5d_3$	1487·7*	7	67218		(5)	
$5p_1—5d_2$	1403·1*	5	71272	4338		97664

* As classified by Carroll.

First Fundamental : $5d_3=97387$
$5d_2=97664$

Classification.	λ I.A.	Int.	ν	Δν	*m*	*mf*
$5d_3$—$4f$	3008·3*	10	33232	281	(4)	64154
$5d_2$—$4f$	2983·04*	8	33513			

Super-Fundamental : $4f=64154$

						mg
$4f$—$5g$	4071·4	6	24554	...	(5)	(39600)

Table B.—Gallium.

First Principal : $4s=247797$

Classification.	λ (vac.)	Int.	ν (vac.)	Δν	*m*	mp_{21}
$4s$—$4p_2$	1495·36*	(12)	66874	1713	(4)	180923
$4s$—$4p_1$	1534·65*	(14)	65161			182636

First Sharp : $4p_2=180923$
$4p_1=182636$

						ms
$4p_2$—$4s$	−1495·36	(12)	−66874	1713	(4)	247797
$4p_1$—$4s$	−1534·65	(14)	−65161			
$4p_2$—$5s$	1353·98*	(4)	75576	1720	(5)	107064
$4p_1$—$5s$	1323·17	(4)	73856			

First Diffuse : $4p_2=180923$
$4p_1=182636$

						md_{32}
$4p_2$—$4d_2$	1295·29*	(1)	77203	109	(4)	103609
—$4d_3$	93·46*	(6)	77312			
$4p_1$—$4d_2$	1267·19*	(6)	78915	1712		103721

First Fundamental : $4d_3=103609$
$4d_2=103721$

						mf_{43}
$4d_3$—$4f$	2424·96*	(7)	41237·8	107·6	(4)	62370
$4d_2$—$4f$	2418·65*	(6)	41345·4			62376

* As classified by Carroll.

Super-Fundamental : $4f_4 = 62370$
$4f_3 = 62376$

Classification.	λ I.A.	Int.	ν (vac.)	Δν	*m*	*mg*
$4f_4$—$5g$	4380·67*	(5)	22821·2	6·2	(5)	(39555)
$4f_3$—$5g$	4381·66*	(5)	22815·0			

* As classified by Carroll.

Second Principal : $5s = 107064$

						mp_{21}
$5s$—$5p_2$	4863·19	...	20556·5	538·5	(5)	86507
$5s$—$5p_1$	4994·15	...	20018·0			87046

Second Sharp Series : $5p_2 = 86507$
$5p_1 = 87046$

						ms
$5p_2$—$5s$	−4863·19	...	−20556·5	538·5	(5)	107064
$5p_1$—$5s$	−4994·15	...	−20018·0			
$5p_2$—$6s$	3806·72	...	26261·9	539·7	(6)	60245
$5p_1$—$6s$	3730·06	...	26801·6			

Second Diffuse Series : $5p_2 = 86507$
$5p_1 = 87046$

						md_{32}
$5p_2$—$5d_2$	3585·44	...	27882·7	62·9	(5)	58561
—$5d_3$	3577·37	...	27945·6	539·0		58624
$5p_1$—$5d_2$	3517·44	...	28421·7			

The spectrograms of Gallium and Indium were found to contain some new lines previously unrecorded, the wave-lengths of which are under measurement ; and the author hopes to give a complete list of wave-lengths for Ga and In in the glass and quartz regions in a future communication.

In conclusion, I gladly take this opportunity of expressing my deep gratitude to Prof. A. L. Narayan for suggesting this problem and for his continued interest and advice throughout the work, and also to Prof. A. Fowler for his interest and encouragement in the pursuit of this investigation.

The author's best thanks are due to Mr. Carroll for having kept him informed of his work before the publication of the Paper in the Trans. Roy. Soc., and also for kindly sending the author a reprint of his Paper soon after its publication.

Explanation of Plates.

Plate I.

Fig. 1 is the spark spectrum of Indium from $\lambda 4700$-$\lambda 3650$; (*a*), (*b*) and (*c*) showing the spectra taken with increasing self-induction.

Fig. 2 is the spectrum of Indium from $\lambda 3500$-$\lambda 2700$; (*a*) and (*b*) representing spectra taken without and with self-induction.

Plate II.

Fig. 1 is the spark spectrum of Gallium in the region $\lambda 4500$-$\lambda 3400$; (*a*) and (*b*) showing the spectra taken without and with self-induction.

Fig. 2 is the spectrum of Gallium from $\lambda 2600$-$\lambda 2300$.

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XII.—ON THE SPECTRUM OF IONISED TIN (Sn III).

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Abstract.

To help in the detection of series in In II attempts have been made to find regularities in Sn III. Visual observations under varying conditions of excitation revealed a strong triplet in the region λ5100. The detection of two more triplets, particularly a diffuse multiplet, confirmed the above identification. A member of the first fundamental series has also been found with the aid of the corresponding members in In II and Ga II. A possible triplet of the first sharp series is suggested, and evidence of its correctness is sought by correlating it with corresponding members of related spectra.

Continuing the work on series in the spectra of doubly-ionised indium and gallium, attempts have been made to discover, if possible, series relations in these spectra corresponding to the singly-ionised stage. By analogy with the spectrum of Al II, it is inferred that in these cases the primary series occur in the extreme ultra-violet, while the principal members of the secondary series fall in the red, and as such do not readily yield themselves for visual examination. Attention is therefore directed, as a preliminary to this, to the detection of series in the spectrum of doubly-ionised tin, which, in general characteristics, should resemble, according to the spectroscopic displacement law, the spectrum of singly-ionised indium.

The spectrum of tin, like that of silicon, is of special interest to spectroscopists, for it must provide another illustration of the possibility of obtaining different types of spectra from a single atom. Occurring in the fourth group of the Periodic Table with silicon, germanium and lead, it has been most refractory as regards the resolution of its spectra into series. In the spark spectrum only the principal doublet of Sn IV has till now been recognised. The present Paper contains a few of the leading members of the spectrum of Sn III, and the investigation of other spectra is in progress.

The experimental procedure adopted is that of examining visually and photographically the spark spectrum of tin in air, in vacuo and in an atmosphere of hydrogen at varying pressures, and also of the " arc in vacuo " between electrodes of tin. To distinguish between lines due to the successive stages of ionisation of the element, the spectrum is observed under different experimental conditions by varying the intensity of discharge, which is done by including in the secondary circuit a variable self-induction and capacity. Great care is taken to make sure that a line belongs to a particular stage of ionisation before it is included into the series system of that stage, and it may be observed that almost all the lines here taken into the series of doubly-ionised tin are found among the list of lines tabulated by Kimura and Nakamura(1) under Sn III, who, by photographing the " cathode spectrum " of tin, have grouped some of the spark lines of tin under successive stages—Sn II, Sn III, Sn IV—as they appear with increasing stimulus.

The notation adopted in the present Paper is that used by Prof. Fowler(2) in " The Spectrum of Ionised Oxygen (O II)," the subscripts denoting the inner quantum numbers. The number denoting the group multiplicity is, however, omitted, as only one system is being considered throughout.

Observations have been made in the visible region, with a constant deviation spectrometer, to find the triplet of the principal (secondary) series which is likely to be observed in this range. The result is the detection of the prominent and intense triplet shown below, and this served as the starting point for the identification of the other members.

Hemsalech.(4) λ	Int.	Eder and Valenta.(5) λ	Int.	Exner(4) and Haschek. λ (I.A.)	Int.	Author. Int.	ν (vac.)	Δν
4858·2	6	4858·4	4	4858·12	5	9	20578·4	
								977·6
5100·8	7	5100·56	3	5100·48	2	8	19600·8	
								465·7
5224·85	5	5225·09	2	5224·53	1	7	19135·1	

The wave numbers are calculated from the accurate measures of Exner and Haschek. The order and magnitude of the intensities indicate that the triplet is inverted, and is, perhaps, the first member of a sharp series. The position of the corresponding triplet of Si III(3) confirmed this suggestion. The separation $\triangle\nu_{p21}$ agrees with that of C III and Si III, as shown.

Atomic number (N).	Element.	Δν	Δν/N^2
6	C III	12·8	0·356
14	Si III	73·2	0·372
50	Ge III	...	...
	Sn III	977·6	0·391

The ratio of the separations 1 : 2·1 is approximately in keeping with Landé's interval rule.

Further examination of the spectrograms, taken with a quartz spectrograph, revealed two more triplets, which, within limits of experimental error, exhibit the same separations. One of these, as is shown below, is accompanied by satellites, and it is therefore inferred that it must be a member of the diffuse series. The region is approximately that where the multiplet *bp*—*bd* is to be expected. It is a group of six lines, and arises from transitions of the electron between triplet *p* and triplet *d* terms. The second is presumably the second member of the sharp series.

The Diffuse Series Multiplet.

Hemsalech. λ	Int.	Hartley and Adeney(4). λ	Int.	Exner and Haschek. λ	Int.	ν (vac.)	Δν
...	...	...	...	...	...	[26715]	
							86·7
3730·2	3	...	...	3730·2	1	26801·7	
							159·7
3708·2	5	3707·6	...	3708·1	2	26961·4	
							976·9
		3609·3	...	3610·3*	...	27691·9	
							86·7
3599·05	2	3598·3	...	3599·02	2	27778·6	
							464·7
		3549·7	...	3550·7	1	28156·6	

* Measured by the writer.

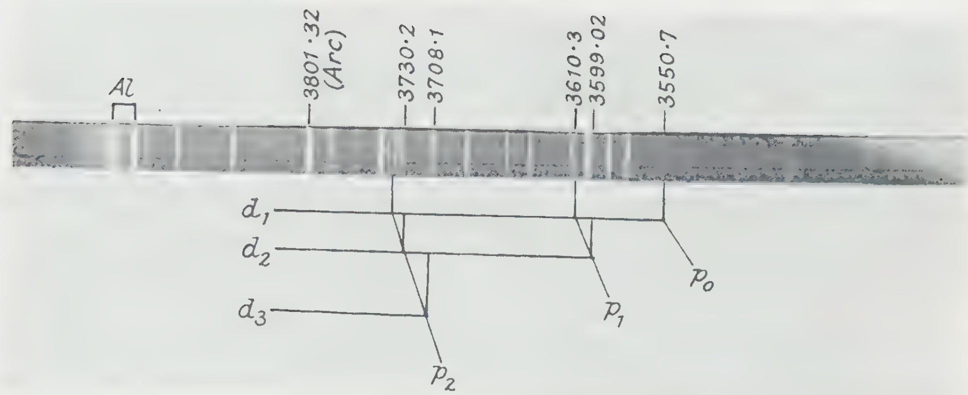

FIG. 1

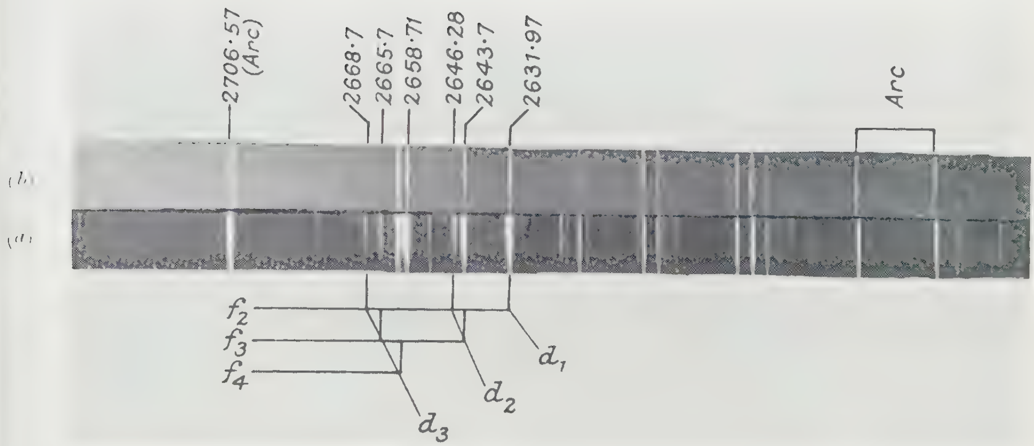

FIG. 2.

FIG. 3

PLATE I.—SPARK SPECTRUM OF TIN

The multiplet differs from that of the member of the diffuse series in Si III inasmuch as the satellites are normal, and appear on the less refrangible side of the main lines, so that the *d* terms are not inverted as in the case of Al II or Si III.

The second triplet is

λ(I. A.)	Int.	ν (vac.)	Δν
2878·5	4	34730·2	
			976·0
2800·6	3	35696·2	
			467·4
2764·4	2	36163·6	

The first line of this triplet has been previously measured by Hartley and Adeney, who record λ2877·4. The limit bp_2 obtained as usual with a 9 N-constant from this member and the first triplet is $\nu=101565$.

Fundamental Series.

The first member of the fundamental series in several spectra is, perhaps, the easiest to fix upon, as the intensity of this member is usually very high, and its position can be approximately located from the values of *d* and *f* terms inferred from a spectral sequence.

There is a very prominent triplet, in the ordinary range of observation, in the spectrum of indium, which from its position, intensity and the value of the separations of the lines is easily seen to be $(ad_{321}-af)$ of In II.

——	λ(6) (I. A.)	Int.	ν	Δν
ad_3-af	4682·00	10	21352·4	
				116 2
ad_2-af	4656·66	10	21468·6	
				82·2
ad_1-af	4638·9	10	21550·8	

The detection of a corresponding narrow triplet in the spectrum of gallium, shown below, substantially supported this assignment.

λ(7)	Int.	ν	Δν
4262·05	10	23456·2	
			34·8
4255·75	9	23491·0	
			25·3
4251·18	9	23516·3	

These led to the identification of the corresponding member in the spectrum of Sn III. In the case of In II and Ga II the *f* levels, presumably triple, are perhaps too close to be resolved with the grating employed, so that a group of only three lines is observable. In the case of tin, however, the *f* levels are also resolved, and

the following six-line multiplet, resulting from the triple d and f levels, is most prominent and is easily detected.

—	ad_3		ad_2		ad_1
af_4	(15) 2658·71 37602·5		... ...		... ...
	98·6				
af_3	(7) 2665·7 37503·9	*312·1*	(15) 2643·7 37816·0		... ...
	36·9		*36·9*		
af_2	(3) 2668·7* 37467·0	*312·1*	(6) 2646·28 37779·1	*205·4*	(15) 2631·97 3798·45

* This satellite which completes the multiplet is measured by the writer.

The Primary Series.

The primary series—the first principal, sharp and diffuse series—fall in the extreme ultra-violet, and attempts are here made to fix these, if possible, first by extrapolation and then to seek for confirmation by correlating the corresponding members of the spectra of related elements. This led to the suggestion of the probable principal triplet of the sharp series. But, in spite of such confirmation, it may be observed that it can only be tentative, and requires experimental confirmation.

The usual Rydberg relation and formula applied to the members of the second sharp series lead to $\lambda 1020$ as the approximate position of the principal triplet $ap_{210}-as_1$, with the larger separation $\triangle\nu_{21}=3200$. A more correct idea of the separation is perhaps obtained from the relation that in the spectra of elements of the same vertical group of the Periodic Table $\triangle\nu/N^2$ is approximately constant. This gives for $\triangle\nu_{21}$ a value of about 4500.

The only available values of wave-lengths of spark lines of tin in the required region are those of Lang.[8]

A careful search was made for a possible triplet among these lines of λ below 1400 A.U., having in view the relative order and magnitude of the intensities and the probable ratio of the intervals between the lines. This suggested the triplet

λ	Int.	ν	$\Delta\nu$
1251·3	60	79917	
			4138
1189·7	40	84055	
			2286
1158·2	60	86341	

Evidence for the possibility of this being regarded as the member $ap_{210}-as_1$ of Sn III is now sought by searching for the corresponding triplet in the spectrum of In II with the aid of the relativistic irregular and regular doublet laws, and also in the spectrum of Ga II, the spectra of gallium being found to resemble very closely the corresponding spectra of indium.

The method as shown below is that adopted by Millikan[(9)] in the identification of lines of C III.

I.—Irregular doublet law, with the usual notation,

$$\frac{\nu'}{R}=\{(n_2^2-n_1^2)Z^2-2(n_2^2\sigma_1-n_1^2\sigma_2)Z+(n_2^2\sigma_1^2-n_1^2\sigma_2^2)\}/n_1^2n_2^2$$

In the case under consideration the line ap_2-as_1 (or using total quantum numbers, $5p_2-6s_1$) is due to an electron jump between two orbits of different total quantum numbers, n_2 being different from n_1.

Hence, transposing

$$\nu'-\frac{R(Z-A)^2(n_2^2-n_1^2)}{n_1^2n_2^2}=C'Z+D'$$

A being a constant introduced for convenience of calculation. The whole expression on the left side varies linearly with the atomic number Z for any particular set of values of n_2, n_1, σ_1, σ_2.

Here $n_2=6$ and $n_1=5$.

$$\therefore\quad \frac{R(n_2^2-n_1^2)}{n_1^2n_2^2}=\frac{R(36\text{-}25)}{36\times25}=1340{\cdot}5$$

The first column in the table below gives the value of ν of Cd I, and the above value tentatively supposed to be that of Sn III.

Putting $A=47$, for Cd I we have the expression on the left equal to $\nu-1340{\cdot}5\times1^2=18316{\cdot}3$

For Sn III

$$=\nu-1340{\cdot}5\times3^2$$
$$=79917-1340{\cdot}5\times3^2=67852$$

At. No.	Element.	$\nu(5p_2-6s_1)$	$[\nu-1340{\cdot}5(Z-A)^2]$	Diff.
48	Cd I	19656·8	18316	
				24415
49	In II	[48093]	[42731]	
				25121
50	Sn III	79917	67852	

Interpolation gives a value for In II of $\nu-1340{\cdot}5\times2^2$, equal to about 43300, or $\nu=48662$.

II.—Regular doublet law: $\{(5p_1-5p_2)=0{\cdot}0234\ (Z-s)^4\}$.

At. No.	Element.	$\Delta\nu\ (5p_1-5p_2)$	$4\sqrt{\Delta\nu/0{\cdot}0234}$	s.
48	Cd I	1171	14·96	33·04
49	In II	[2481]	[18·05]	[30·95]
50	Sn III	4138	20·5	29·5

Interpolation gives about 31 for the screening constant " s " for In II, which leads to

$$\triangle \nu = 0{\cdot}0234(49-31)^4$$
$$= 2457.$$

The following strong triplet in the spectrum of indium is found to be exactly in agreement with the calculated values. The respective values, adopting this triplet, are shown in the above tables enclosed in brackets. It is seen that these laws are exactly satisfied. For purposes of comparison, the values of λ and intensity by different observers are given below.

Triplet ($5\,p-6s$) *of In II.*

Saunders.(10) λ	Int.	Weinberg.(11) λ	Int.	Eder & Valenta.(4) λ	Int.	Carroll.(7) λ	Int	ν (vac.)	$\Delta\nu$
2079·28	7	2079·6	5	2078·8	8	2079·3	4	48093	
									2481
1977·44	5	1977·8	6	1976·8	7	1977·3	3	50574	
									1058
		1936·5	4	1935·9	5	1936·8	3	51632	

The wave-numbers are calculated from the recent accurate measurements of Carroll. The exact coincidence of observed and calculated values of this triplet perhaps lends good support to the correctness of the identification.

Further support is given by the detection of the corresponding triplet, shown below, in the spark spectrum of gallium, which is very prominently seen in the beautiful spectrograms accompanying Carroll's Paper, and agrees excellently with that of In II as regards position, frequency, interval, etc.

Triplet ($5p-6s$) *of Ga II.*

Weinberg. λ	Int.	Carroll. λ	Int.	ν (vac.)	$\Delta\nu$
1845·0	8	1845·28	9	54201	
					932
1813·8	9	1813·91	9	55133	
					450
1799·1	6	1799·31	7	55583	

There is a displacement of the line towards the region of shorter wave-length as we pass from In II to Ga II. Values of $\triangle\nu/N^2$ for the elements of the same vertical group Al II, etc., are shown in the following table.

At. No. N.	Element.	$\Delta\nu\ (5p_1-5p_2)$	$\Delta\nu/N^2$
13	Al II	125·5	0·743
31	Ga II	932·0	0·97
49	In II	2,481·0	1·034

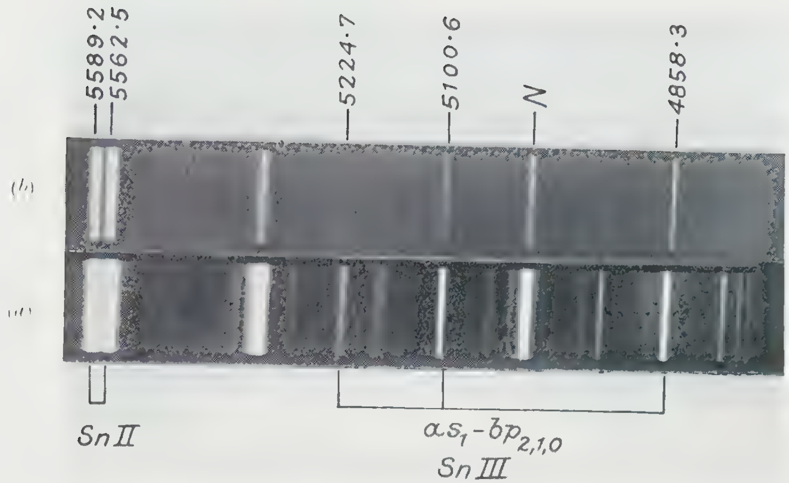

Fig. 4.

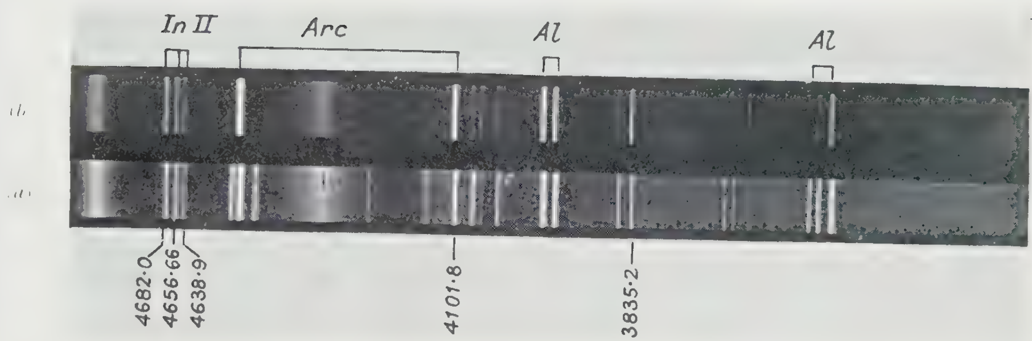

Fig. 5

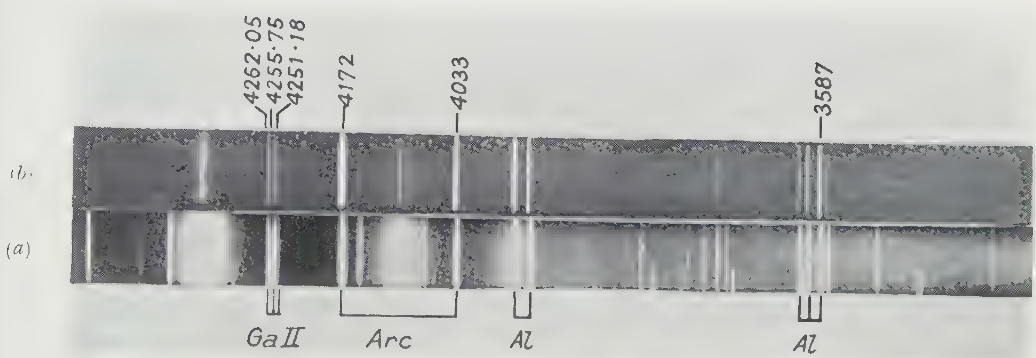

Fig. 6

Plate II Fig. 4 Spark Spectrum of Tin.
Fig. 5. „ „ „ Indium.
Fig. 6 „ „ „ Gallium.

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For comparison, a similar table for the group of spectra Al III, etc., is also added.

N.	Element.	$\Delta\nu$	$\Delta\nu/N^2$
13	Al III	238	1·408
31	Ga III	1714	1·7
49	In III	4347	1·81

Term Values.—The frequency values of the various energy levels in the spectrum of doubly-ionised tin cannot be determined at this stage. For the first member of the first diffuse series ($ap-ad$) has not yet been located for want of accurate data of the spark spectrum of tin in the region of short waves below about λ1000 A.U., and this alone would give, with the aid of the above determined members, a connected system of terms from af to ap.

Spectrum of Sn II.

It may not be out of place here to include two pairs of the same separation which are found in the spark spectrum of tin and definitely belong to the doublet system of Sn II.

λ	Int.	ν (vac.)	$\Delta\nu$
3352·47	10	29821·5	625·4
3283·6	9	30446·9	
2488·0	5	40180·6	626·3
2449·9	4	40806·9	

It is difficult now to conclude definitely about the series to which these pairs belong. But the lines are very intense and diffuse under all the conditions examined, except in the "arc in vacuo," where they appear with great sharpness. And no satellite could be detected accompanying any of these lines.

Applying the usual Rydberg formula with a four-fold constant, they give a limit equal to 58232 and term values 28408 and 18048. These latter suggest that the members probably form a combination series of the type $x_{12}-mf$, a series of exactly this nature being found in the spectrum of Si II by Prof. Fowler (loc. cit.).

Explanation of Plates.

Plate I.

Fig. 1 indicates the spectrum of tin showing the diffuse multiplet at about λ3700.

Fig. 2 shows the prominent six-line multiplet of the first fundamental series; (a) and (b) are photographs taken without and with inductance in the secondary circuit.

Fig. 3, the spectrum (spark) of tin in the region λ2500, showing the pair λ2488 and 2449 of Sn II.

Plate II.

Fig. 4, (a) and (b), spark spectra between tin electrodes without and with inductance respectively.

Figs. 5 and 6, spectra of indium and gallium indicating the triplets of the first fundamental series of In II and Ga II respectively ; (*a*) and (*b*), as before, are photographs taken without and with inductance respectively.

In all these the metal is contained in Al cups.

In conclusion, I wish to express my great indebtedness to Prof. A. L. Narayan, M.A., D.Sc., F.Inst.P., for giving me valuable guidance throughout the course of the investigation.

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DEMONSTRATION OF THE BEHAVIOUR OF BODIES WITH NON-CONDUCTING SURFACES IN ELECTROSTATIC FIELDS.

By L. G. VEDY.

THE rotation of a piece of dielectric material when suspended between the poles of a Wimshurst machine was observed, about a year ago, by S. W. Richardson, Esq., M.A., D.Sc., F.Inst.P., Chief Physics Master at Whitgift Grammar School, Croydon, who kindly allowed me to investigate this phenomenon in the school laboratory. The experiments which I have performed up to the present, in which I have been materially assisted by Mr. G. Gowlland, indicate, in general, the following results :—

1. Any body consisting of or covered with dielectric material, and free to turn about an axis, rotates when placed between the poles of a Wimshurst machine. This effect increases with the thickness of the surface layer to a certain extent, and is stopped if the body is covered with conducting material.

2. Rotation occurs only when a discharge takes place from either or both of the poles of the machine. It occurs in either uniform or non-uniform fields, increases with the amount of the discharge, masking the orientation effect of dielectrics, and usually occurs in either direction. If, however, the knobs are pointed tangentially towards the surface rotation proceeds more easily in the direction in which they point. If, instead of knobs, points are used the rate of rotation is much less, but the directive effect is more marked.

3. The effect also occurs, but to a much less extent, with partial conductors (e.g., cork) if there is sufficient discharge present. Also in the case of a body covered with a thin dielectric surface layer, the rate of rotation is greater when the interior is a poor conductor (e.g., cork) than when it is a metal.

4. In general a body is charged during rotation. The charge is the same kind as that of the pole from which it receives the greater discharge. When mounted systematically between two similar knobs the body has no detectable charge.

5. These effects are completely stopped if the body is shielded by a thin plate of dielectric substance. The other effects, however, of electrostatic fields such as orientation of dielectrics, etc., are unaffected.

The phenomenon appears to be due to the effect of the discharge on the non-conducting surface layer of the body, and does not appear to be associated with the nature of the dielectric (as are the orientation effects, etc.), and the experiments performed seem to show that local charging of part of the surface and the subsequent repulsion of this maintain the rotation.

These effects were illustrated by the following experiments :—

Pieces of paraffin wax, ebonite, sealing wax, and a glass beaker rotated when hung from threads between the knobs of a Wimshurst machine, but not when covered with tin or copper foil. Brass spheres, etc., covered with paraffin wax, rotated, the greatest rate being attained by the sphere with the thickest wax layer. Better results were obtained with bodies mounted on bearings (steel needles in glass sockets),

as this prevented the body from swaying and knocking the knobs. Rotation of a paraffin sphere occurred about any axis, provided this did not make a large angle with the plane perpendicular to the line joining the centres of the knobs.

By using an alternative gap it was shown that rotation did not occur in the absence of a brush discharge. Also sparking usually decreased the effect. The directive effect of knobs and points was shown by the direction in which the rotation of a paraffin wax sphere (about 10 cms. diameter) started, when the knobs or points were directed tangentially towards it.

In the presence of a large brush discharge a cork (about 10 cms. diameter, and mounted on bearings) rotated. When covered with paper the rate of rotation was increased, though it was considerably decreased if a layer of tinfoil was placed under the paper.

A cardboard tube covered partially with paraffin wax commenced rotation in such a direction that the paraffined portions moved away from the knobs, thus indicating a repulsive effect.

The presence of a charge on the bodies was shown by using a proof plane and an ordinary gold-leaf electroscope. When the paraffin sphere was mounted symmetrically between two nearly similar knobs the charge was shown to be very small.

When the bodies were suspended inside a glass beaker, or between ebonite plates, which shielded them from the discharge, rotation did not occur, but the ordinary orientation effects were obtained. This was confirmed by the charges found to be present on the parts of the body near to the knobs. In particular, a sphere remained in the position in which it was originally placed, and if displaced oscillated about this position. If, however, a charge was communicated to it, by means of a conductor touching one of the knobs, rotation commenced.

DISCUSSION.

Prof. F. L. Hopwood (communicated): I should like to call the attention of those interested to a record of similar experiments to those shown, which is contained in the chapter on Electrostatic Motors in V. E. Johnson's book "Modern High Speed Influence Machines" (Spon, 1921).

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